Analysis of Small-Scale Hydraulic Systems

Jicheng Xia and William K. Durfee

Department of Mechanical Engineering, University of Minnesota
111 Church Street SE, Minneapolis, MN, 55455
Correspondence to Durfee, 612-625-0099, wkdurfee@umn.edu

***Abstract*— We investigated small-scale hydraulic power system using a system level analysis, where small-scale refers to systems generating 10 to 100 Watts output power, to determine whether the high power density advantage of hydraulic power system holds at small sizes. Hydraulic system power density was analyzed with simple mathematical models and compared to an equivalent hypothetical electromechanical system constructed from high-end off-the-shelf components. Calculation results revealed that high operating pressures are needed for small-scale hydraulic power systems to be lighter than the equivalent electromechanical system.**

***Index terms* — small scale hydraulics, electromechanical, fluid power, power density, system level analysis.**

I. Introduction

Hydraulic fluid power systems are well known for their high power density [15], [3]. This advantage is best illustrated in applications such as excavators and heavy manufacturing equipment that require extremely large power and force. Hydraulics is the only practical way to attain these levels of force and power while at the same time being relatively light weight compared to the equivalent electromechanical system. One reason for the high power density of hydraulics is that fluid power cylinders are inherently low-velocity, high-force actuators, which is a good match to the requirements for construction, agricultural and manufacturing heavy equipment. Contrast this with electric motors that are high-velocity, low torque actuators and require a transmission such as a gear head or a lead screw to match their optimal operating point to the application. At high forces and torques, the weight of the transmission ends up being a significant fraction of the actuator package weight. A second reason for the high power density is that exceptionally high pressures can be generated (excavators run up to 5000 psi).

An advantage of hydraulics is that the source of pressurized fluid can be housed in a base station and flexible hoses used to transport the fluid to light weight cylinders located at the periphery of the machine. For example, an excavator has actuators to control the boom, stick and bucket with bulky power supply, reservoir and accumulators placed in the house. The more proximal actuators carry the load of the more distal. When the excavator arm is fully extended, the bucket actuator at the end of the arm causes large moments at the joint connecting the boom to the house, which requires a powerful boom actuator. If the bucket actuator is a cylinder, the weight of the actuator is small compared to the bucket. If the bucket actuator is electromechanical, the weight of the electric motor and its associated transmissions, both of which must be placed at the joint, can be significant.

The power density of electromechanical systems has an upper limit because of inherent characteristics such as magnetic saturation. In contrast, the power density of hydraulic systems has no inherent upper limit and can be increased by increasing the pressure. The main barriers for increasing power density in a hydraulic system are being able to design the containing structure and the seals.

There has been recent interest in portable, wearable powered systems including powered exoskeletons and powered orthotics [2], [21]. Examples of mobile systems in the 10 to 100 W range include ankle foot orthotics, small robots and powered hand tools. These devices are usually powered by electromechanics, typically a lithium-ion battery, DC electric motor and transmission. Little work has been done on using hydraulics for these applications because off-the-shelf tiny hydraulic components (cylinders, valves, power supplies) do not exist.

Designers may wish to consider hydraulics for tiny, mobile powered systems because it would seem that the same power density advantage of hydraulics over electromechanical should hold for a powered orthosis as it holds for an excavator. The story, however, is complex because the scaling laws are not intuitive. For example, in a cylinder, force is proportional to area (L^2) while weight is proportional to volume (L^3). Surface effects such as friction drag of seals and viscous drag of gaps become significant at small bores and impact overall efficiency. On the other hand, the thickness, and thus the weight, of a cylinder wall required to contain a fixed pressure goes down with bore. The final weight of a hydraulic system at small scale cannot be determined by proportionally scaling the weight of a large system. Thus the answer to the question, "For equal efficiency, which is lighter a fluid power or an electromechanical system?" for a tiny system cannot be answered using intuition or simple scaling.

Love [9] demonstrated an application of small scale hydraulics by prototyping a prosthetic finger. Pressure as high as 2000 psi (about 13.8 MPa) was used to operate 4 mm hydraulic cylinders. In this way, much higher power density was achieved by hydraulic cylinder than electric motor. Another example of using small scale hydraulics is illustrated in endoscope platform[14]. Two different systems were tried, hydraulics and electric. It was proven that hydraulic system can provide larger output force in limited space.

One of the challenges for increased hydraulic power density is the seals. Volder et al. developed new seals for microactuators, such as ferrofluid seal [19] and PDMS seal [18]. 1.6 MPa (about 230 psi) pressure was sealed without leakage using these seals. To generate pressurized fluid, a hydraulic pump is needed. Small scale hydraulic pumps exist in off-the-shelf components and laboratory prototypes. One small commerical pump [13] can operate at 4000 psi pressure, 27 cc/s and 5000 rpm shaft speed with a size of 3.3 cm dia. x 5.1 cm long. The overall efficiency of the pump, however, is only 30%. Shin [16] prototyped a small scale hydraulic pump that uses thin film shape memory alloys (SMA). The prototype produced 7 cc/s flow rate and 200 N blocking force. A unique feature of the pump is that hydraulic fluid was used to cool the SMA film, which significantly increased the actuation frequency of the membrane. Microfluidics has made tremendous advances in recent years, but cannot inform our problem as microfluidic systems operate well under 1 W and our systems of interest are in the range of 1 to 10 W. Reviews of microfluidics components are given in [8], [12] and [20]. As shown in [20], micro fluid power cylinders in the research lab can generate 1 to 10 N but with strokes under 1 mm .

The aim of this study was to use first principles to understand how the weight and other properties of hydraulic systems change with size and to answer the question, "Is a hydraulic solution lighter than an electromechanical solution for tiny, powered systems?" Our goal was to provide guidelines that mechanical designers could use at the early stages of evaluating architectures for small systems. Empirical and analytical equations were used to model hydraulic and electromechanical systems, connecting the methods to real components wherever possible.

Because the problem is complex, our approach was to conduct an analysis for a idealized, linear actuation benchmark system. The analysis only considers the transmission line and actuator, leaving the power supply and control means for future work. Because the supply and control can be sited anywhere on the machine, this limitation, while important, does not diminish the utility of the study. The results of the analysis shows that, for equal output power and system efficiency, a hydraulic solution will be lighter than an electromechanical solution only if the hydraulics operates at high pressure.

II. Benchmark System

The top row of Fig. 1 illustrates the architecture of a generic mobile actuation system that contains a power supply, a means of control, a transmission line and an actuator located at the end-point. For this study, we considered systems that delivered force and velocity along a linear axis. For example, this includes a powered knee prosthesis with the joint driven by a linear actuator mounted behind the knee. Or, it includes a powered gripper with a mechanism driven by a linear actuator.

The electromechanical realization (middle row of Fig. 1) includes a battery power supply, a PWM motor controller, wire, a brushed or brushless DC electric motor and a lead screw or ball screw to convert the high velocity, low torque output of the motor to a low velocity, high force linear output. The ball screw was chosen because it is lighter and more efficient than the equivalent gear box, and it converts rotary to linear motion, which provides a fair comparison to the hydraulic system. The hydraulic version (bottom row of Fig. 1) includes a battery or internal combustion engine driven pump to generate pressured fluid, a servovalve, pipe or hose and a hydraulic cylinder. Other realizations are possible.

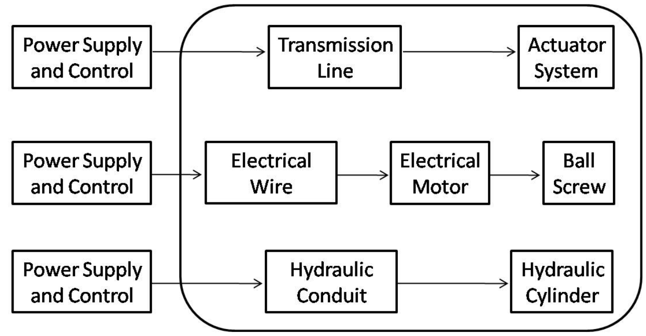

Fig. 1. Architcture for powered actuation system. Top row is generic, middle row is electomechanical, bottom row is hydraulic.

This study only considers the transmission line plus actuator system, the circled components in Fig. 1. These are the parts of the system that must be located at the point of mechanical output where weight is of greatest concern. For example, for a portable hand tool, the power supply and control can be placed in a backpack or tool belt, but the transmission line and actuator system must be held in the hand. In a real mobile system, the power supply will contribute significantly to the weight and in a real system, the control means will contribute significantly to the efficiency. Comparing electromechanical and hydraulic endpoint components, however, still provides valuable information to the designer looking to minimize weight at the endpoint.

III. Hydraulic System Analysis

The objective of the hydraulic system analysis was to estimate the weight of an ideal hydraulic cylinder plus the weight of ideal conduit to predict the total weight for a hydraulic system that delivers a specified mechanical force and power output. The weight of components was estimated from a set of theoretical equations developed using basic physics of fluids and solid mechanics.

A. Hydraulic Cylinder

The simplified hydraulic cylinder used for analysis is illustrated in Fig. 2 and its associated parameters are defined in Table I. The cylinder is double-ended with bore B, stroke S and rated maximum pressure P_r. The piston is a disk of uniform thickness t_1 and the cylinder housing is a capped tube with barrel wall thickness t_2 and end cap thicknesses t_3 and t_4. For simplicity, O-ring seals are assumed for piston and rod. Only uni-direction extension motion is considered with cap side pressure P_1 and rod side pressure zero.

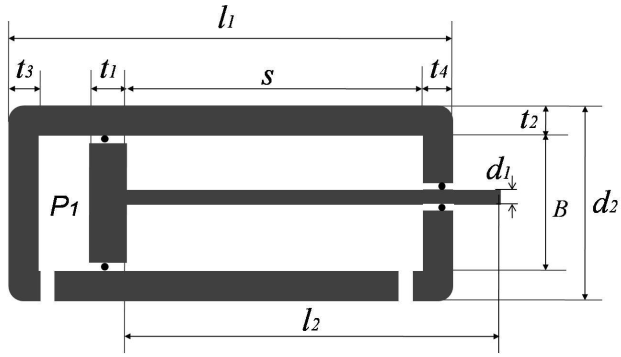

Fig. 2. Ideal hydraulic cylinder used for analysis.

TABLE I
HYDRAULIC CYLINDER PARAMETERS.

VAR	*DESCRIPTION*	*UNIT*
B	bore	m
S	stroke	m
l_1	cylinder length	m
l_2	rod length	m
t_1	piston thickness	m
t_2	cylinder circumferential wall thickness	m
t_3	left end wall thickness	m
t_4	right end wall thickness	m
d_1	rod diameter	m
d_2	outer diameter	m
P_m	maximum allowable fluid pressure	Pa
P_1	cylinder left chamber pressure	Pa
S_y	cylinder material yield strength	Pa
E	cylinder material Young's modulus	Pa
ρ	cylinder material density	Kg/m^3
ν	cylinder material poisson's ratio	—
N	design safety factor	—

1) Cylinder and Piston Wall Thickness: The pressure loading scenario to calculate the required cylinder wall and the piston thickness is shown in Fig. 3 where the cylinder rated pressure P_r acts everywhere on the wall. The end wall calculations assumed a fixed displacement boundary condition along the end wall circumference. The piston thickness calculation assumed that the rod was fixed and the P_r was distributed uniformly across the cap side of the piston and zero pressure on the rod side. These are all worst-case loading conditions.

The cylinder circumferential wall thickness was calculated using the equation for a thin-walled pressure vessel [4]

$$t_2 = \frac{N \cdot P_r \cdot B}{2S_y} \tag{1}$$

which is valid for $t_2 < B/6$. The cylinder end wall thicknesses, t_3 and t_4, and the piston thickness t_1 were calculated using thin plate formulas [11]

$$t_1 = \sqrt{\frac{3NP_rG_1\nu}{4S_y}} \tag{2}$$

$$t_3 = \sqrt{\frac{3\pi B^2NP_r(1+\nu)}{32\pi S_y}} \tag{3}$$

$$t_4 = \sqrt{\frac{3NP_rG_2}{4\nu S_y}} \tag{4}$$

where G_1 and G_2 are defined as

$$G_1 = \frac{4B^4(1+\nu)\log\frac{B}{d_1} + 4\nu B^2d_1^2 + d_1^4(1-\nu) - B^4(1+3\nu)}{4\nu(B^2-d_1^2)}$$

$$G_2 = \frac{d_1^4(1-\nu) - 4d_1^4(1+\nu)\log\frac{B}{d_1} + B^2d_1^2(1+\nu)}{4B^2(1-\nu)+4d_1^2(1+\nu)} + \frac{B^2}{4} - \frac{d_1^2}{2}$$

The thin plate formulas are valid for plate thickness that are less than $1/4$ of the plate diameter. The formula used to determine t_4 was that for a round plate containing a central hole.

2) Rod Diameter: The rod must be sized so that it will not buckle under the maximum compressive load. The required rod diameter was calculated using Euler and JB Johnson buckling formulas [11], assuming that the rod was fully extended, loaded in compression and carrying the piston force at the maximum rated pressure. The slenderness ratio $\frac{l_2}{\rho_1}$ dictates whether the Euler or the JB Johnson formula is appropriate. The critical rod slenderness ratio is

$$\left(\frac{l_2}{\rho_1}\right)_{\text{crit}} = \sqrt{\frac{2\pi^2E}{S_y}} \tag{5}$$

where $\rho_1 = d_1/4$ for a solid round rod. For a slenderness ratio less than the critical value the JB Johnson formula was used

$$d_1 = \sqrt{\frac{4NP_r\pi(B/2)^2\eta_f}{\pi S_y} + \frac{4S_yl_2^2}{\pi^2E}} \tag{6}$$

and for other cases, the Euler formula was used

$$d_1 = \left(\frac{64Nl_2^2P_r\pi(B/2)^2\eta_f}{\pi^3E}\right)^{\frac{1}{4}} \tag{7}$$

The cylinder force efficiency η_f, which results from sealing friction, is calculated in the next section.

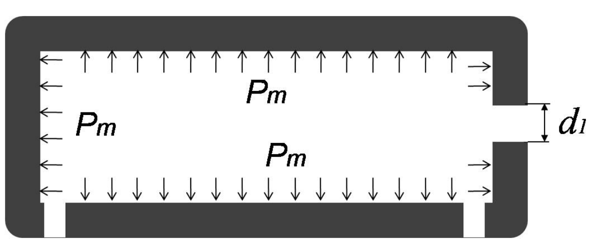

Fig. 3. Wall loading scenario used to calculate wall thickness

TABLE II

SYMBOLS USED IN EQUATIONS (9) AND (10)

VAR	*DESCRIPTION*	*UNIT*
F_s	friction force piston with seal	N
f_s	O-ring seal friction coefficient	—
D	piston or rod diameter	m
d	O-ring cross-sectional diameter	m
E_s	O-ring Young's modulus	Pa
ϵ	O-ring squeeze ratio	—
Q_s	leakage across sealed piston or rod	m^3/s
μ	hydraulic fluid dynamic viscosity	Pa· s
U_{hc}	piston velocity	m/s
δ_m	maximum O-ring contact stress	Pa
s_0	O-ring contact width	m

3) Cylinder Efficiency: The force in the rod is less than the pressure times the area of the piston because of the friction in the piston and rod seals. The cylinder force efficiency, η_f is defined as

$$\eta_f = \frac{F_r}{P_1 A_1} \tag{8}$$

where F_r is the rod compressive force, P_1 is the cap side pressure and A_1 is the cap side piston area [10].

Equations (9) and (10) are approximations that describe the seal friction [1] and leakage ([6], [5]) for a rubber O-ring seal. Variables for these equations appear in Table II.

$$F_s = f_s \cdot \pi \cdot D \cdot d \cdot E_s \cdot \epsilon \cdot \sqrt{2\epsilon - \epsilon^2} \tag{9}$$

$$Q_s = 2.99 \cdot \pi \cdot D \cdot \mu^{0.71} \cdot U_{\mathrm{hc}}^{1.71} \cdot \delta_m^{-0.71} \cdot s_0^{-0.29} \tag{10}$$

Applying equations (9) and (10) for the piston and the rod yields the estimation of the cylinder force efficiency, volumetric efficiency and overall efficiency

$$\eta_f = \frac{P_1 A_1 - F_{sp} - F_{sr}}{P_1 A_1} \tag{11}$$

$$\eta_q = \frac{V_r A_1}{V_r A_1 + Q_{sp} + Q_{sr}} \tag{12}$$

$$\eta_{\mathrm{hc}} = \eta_f \cdot \eta_q \tag{13}$$

where F_{sp} is piston seal friction force (Newton), F_{sr} is rod seal friction force (Newton), V_r is rod velocity (meter/sec), Q_{sp} is piston seal leakage (cu-m/sec) and Q_{sr} is rod seal leakage (cu-m/sec).

4) Cylinder Weight: The volume of the cylinder is

$$V_{\mathrm{cyl}} = \frac{\pi}{4}\Big[(d_2^2 - B^2)l_1 + B^2(t_3 + t_1 + t_4) + d_1^2 l_2 - \Delta_V\Big] \tag{14}$$

where Δ_V are the adjustments to the volume due to the inlet, outlet and rod openings. For simplicity, only the rod opening volume will be included as the inlet and outlet openings will be balanced by the volume of fittings.

$$\Delta_V = d_1^2 \cdot t_4 \tag{15}$$

Assuming the same material is used for the cylinder wall, piston and rod, the weight of the cylinder is

$$M_{\mathrm{cyl}} = \rho \cdot V_{\mathrm{cyl}} \tag{16}$$

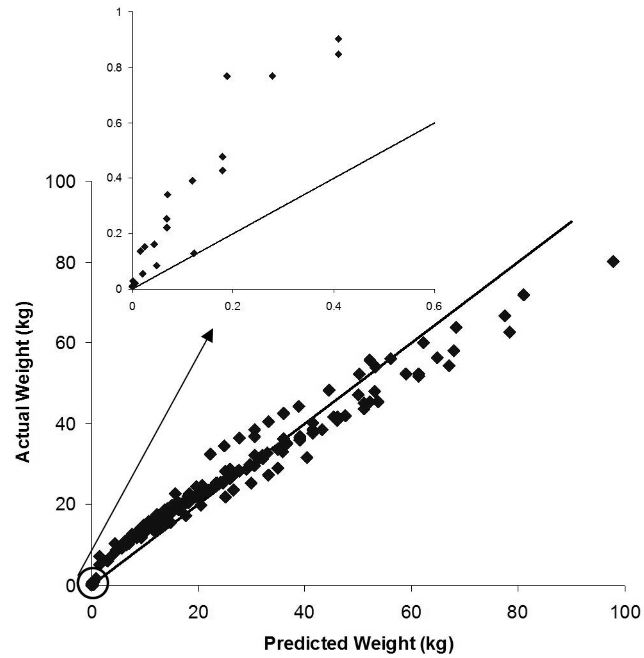

Fig. 4. Comparison between the actual weight and the weight predicted from the theoretical analysis for 187 commercial cylinders. The solid line indicates an exact match between actual and predicted. The inset expands the data for light weight cylinders.

5) Validation: To validate the calculation of estimated cylinder weight based on the basic theory presented in the previous sections, (16) was used to predict the weight of commercially available cylinders. Catalog data for 187 hydraulic cylinders from from several manufacturers (Airpot, Beily, Bimba, Hercules, Prince) was used to build a database of rated pressure, bore, stroke and weight for real products. For the analysis, the cylinder material was assumed to be 304 stainless steel, which provided the yield strength, Young's modulus, Poisson's ratio and material density for the equations. (A real cylinder would be fabricated from several materials.) The safety factor N was set to 2 as this was the value found in two of the vendor catalogs. Common parameters were used for O-ring seal and hydraulic oil: 10% for squeeze ratio, 10 MPa for O-ring Young's modulus, 1 mm for O-ring seal cross-section diameter and 0.1 Pa·s for fluid viscosity. The pressure, bore, stroke, material properties and safety factor were used to calculate the theoretical wall thickness, volume and weight for the cylinder. The theoretical weight was then compared to the real catalog weight for the cylinder.

Fig. 4 compares the predicted weight to the actual weight. If the theory matched the real cylinders exactly, all data points would lie on the solid line. Practical cylinders are somewhat lighter than their predicted weight for heavier cylinders, and somewhat heavier than their predicted weight for lighter cylinders (see the figure inset).

A test to check whether the assumptions underlying certain formulas used were valid was run on the 187 cases. The assumptions tested were: (1) cylinder wall thickness smaller than $1/6$ of cylinder bore for correct use of the thin-walled cylinder formula [4]; (2) plate thickness smaller than $1/4$ of plate diameter for correct use of the thin plate formula [11]; (3) rod diameter smaller than cylinder bore to satisfy physical feasibility. With a safety factor of two, some of the theoretical results violated the thin plate formula assumption for the piston and the rod end cylinder wall. This can be corrected by using a safety factor of less than two, which would result in thinner pistons and walls. The formula assumptions held for the 20 lightest cylinders in the data base with weights less than 0.4 kg.

B. Hydraulic Conduit

For smooth pipes, the fluid flow equations [22] are

$$P_2 - P_1 = \frac{f_\mathrm{p} \cdot \rho_\mathrm{f} \cdot V_\mathrm{p}^2 \cdot L_\mathrm{p}}{2 \cdot D_\mathrm{p}} \tag{17}$$

$$A_\mathrm{p} = \frac{\pi \cdot D_\mathrm{p}^2}{4} \tag{18}$$

$$V_\mathrm{p} = \frac{Q_\mathrm{p}}{A_\mathrm{p}} \tag{19}$$

$$Re = \frac{\rho_\mathrm{f} \cdot D_\mathrm{p} \cdot V_\mathrm{p}}{\mu} \tag{20}$$

$$f_\mathrm{p} = \begin{cases} 64/Re & \text{laminar flow} \\ 0.316/Re^{0.25} & \text{turbulent flow} \end{cases} \tag{21}$$

where P_2 is pipe inlet pressure (Pascal), P_1 is pipe outlet pressure (Pascal), f_p is pipe friction coefficient, ρ_f is fluid density (kg/cu-m), V_p is pipe flow velocity (m/s), L_p is pipe length (m), D_p is pipe inner diameter (m), A_p is pipe cross-section area (sq-m), Q_p is pipe flow rate (cu-m/sec) and Re is the Reynolds number.

Using equation (17) through equation (21), the pipe efficiency is

$$\eta_\mathrm{p} = \frac{P_1}{P_2} \tag{22}$$

$$= \begin{cases} 1 - \frac{128\mu}{\pi} \cdot \frac{Q_\mathrm{p} \cdot L_\mathrm{p}}{D_\mathrm{p}^4 \cdot P_2} & \text{laminar} \\ 1 - \frac{1.79\mu^{0.25} \cdot \rho_\mathrm{f}^{0.75}}{\pi^{1.75}} \cdot \frac{Q_\mathrm{p}^{1.75} \cdot L_\mathrm{p}}{P_2 \cdot D_\mathrm{p}^{4.75}} & \text{turbulent} \end{cases}$$

These equatiions enable calculating the pipe i.d. D_p as a function of $\mathrm{Q_p}$, L_p, P_2 and η_p.

The pipe weight can be calculated once the pipe wall thickness is found using the thin-walled cylinder formula [4]

$$t_5 = \frac{N \cdot P_2 \cdot D_\mathrm{p}}{2 S_y} \tag{23}$$

where t_5 is wall thickness (m), N is design safety factor, and S_y is pipe material yield strength (Pa). For this analysis we assumed that the pipes, like the cylinders, were fabricated from 304 stainless steel.

The weight of the pipe is

$$M_\mathrm{conduit} = \pi\Big((\frac{D_p}{2} + t_5)^2 - (\frac{D_p}{2})^2\Big) L_p \rho_p \tag{24}$$

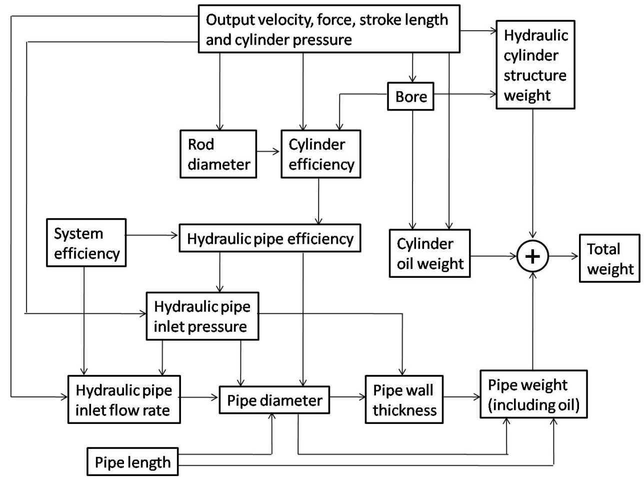

Fig. 5. Method for calculating the weight of a hydraulic system.

where ρ_p is the pipe density (kg/cu-m). The weight of the oil in the pipe is

$$M_\mathrm{ConduitOil} = \pi(\frac{D_p}{2})^2 L_p \rho_f \tag{25}$$

where ρ_f is the oil density (kg/cu-m).

The pipe efficiency η_p, inlet pressure P_2 and fluid flow rate Q_p were calculated using

$$\eta_\mathrm{p} = \frac{\eta_\mathrm{sys}}{\eta_\mathrm{cyl}} \tag{26}$$

$$P_2 = \frac{P_1}{\eta_\mathrm{p}} \tag{27}$$

$$Q_\mathrm{p} = \frac{F_r \cdot V_r}{\eta_\mathrm{sys} \cdot P_2} \tag{28}$$

where η_sys is the desired overall efficiency (Section IV), η_cyl is the cylinder efficiency, F_r is rod force (N) and V_r is rod velocity (m/sec).

C. Hydraulic System Weight

Fig. 5 illustrates how the weight of the hydraulic system is calculated. First, the output force, output velocity and stroke length are specified by the application requirements. Using this information, cylinder weight, efficiency, bore and rod diameter are calculated. Next, using the overall system efficiency of the equivalent electromechanical system (Section IV-D), the hydraulic pipe inlet power and efficiency is calculated. Hydraulic pipe inlet pressure is calculated for a given cylinder operating pressure. Next hydraulic pipe inlet flow rate is calculated using inlet power and pressure, then the hydraulic pipe diameter using inlet pressure, inlet flow rate, pipe efficiency and pipe length information, as shown in equation (22). With these numbers, pipe weight is calculated. Finally, total system weight is calculated by summing weights of the cylinder, pipe and hydraulic oil contained in the cylinder and pipe.

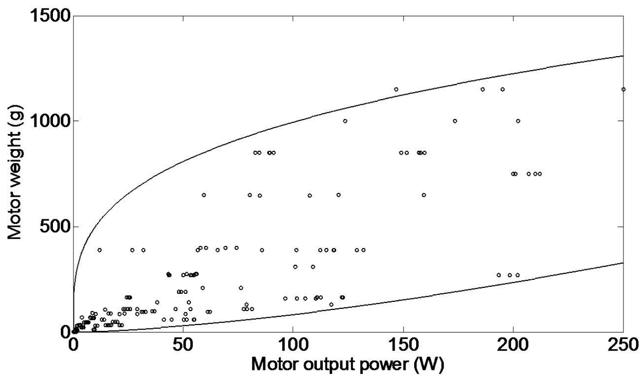

Fig. 6. Motor weight vs. output power

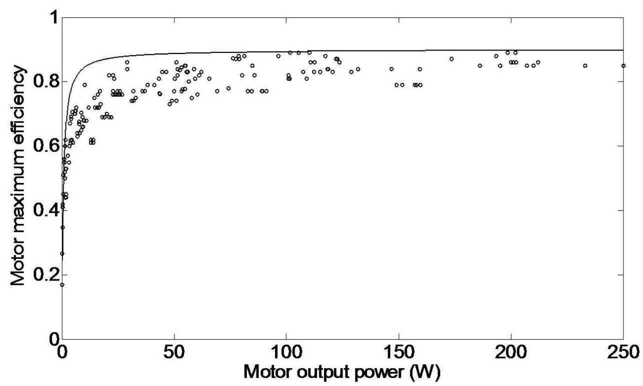

Fig. 7. Motor efficiency vs. output power

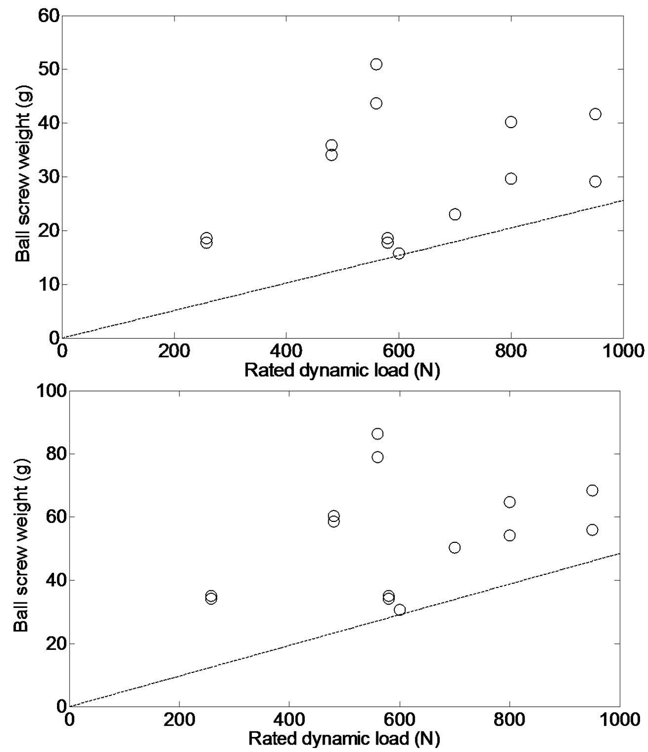

Fig. 8. Ball screw weight vs. rated dynamic load at .01 m (top) and .04 m (bottom) stroke.

IV. Electromechanical System Analysis

The electromechanical system includes wire for the transmission line, a DC electric motor and a ball screw. Unlike hydraulic components, electromechanical components for small-scale applications are readily available. Therefore, rather than using theoretical methods, the approach to estimating the total weight of an electromechanical solution was to develop a set of empirical equations that captured the scaling of component weight and efficiency with load or power based on the properties of high-end, commercially available electromechanical components captured from company catalogs.

A. DC Electric Motor

The key system-level parameters for DC electric motors are weight and efficiency. Brushless, permanent magnet DC motors were chosen because for small precision applications they have the highest efficiency and highest power density. Power, weight and efficiency data for 192 motors from two manufacturers (MicroMo Electronics Inc. and Maxon Motor) were collected. The power for a motor was taken as the peak continuous mechanical output power and the efficiency was the electrical power in to mechanical power out maximum efficiency at the nominal voltage. Fig. 6 plots motor weight versus motor power and Fig. 7 plots motor efficiency versus power.

For modeling purposes, empirical equations were created to bound motor properties. The lower curve in Fig. 6 is the lower bound of motor weight. Using this curve in a system analysis means that one is looking for the lightest commercially available motor for a given power. The upper curve in Fig. 7 is the upper bound of motor efficiency. Using this curve in an a system analysis means that one is looking for the highest efficiency motor for a given power. The two bounding curves are

$$W_m = \frac{P_m^{1.5}}{12} \quad (29)$$

$$E_m = 0.9 - 0.9 \cdot \frac{0.1}{0.15 \cdot P_m + 0.1} \quad (30)$$

where W_m is motor weight, η_m is motor efficiency and P_m is motor power (W).

B. Ball Screw

The ball screw converts the motor rotary power to low speed, high force linear power. The weight of a ball screw is related to its rated dynamic load and to its stroke length. Weight does not depend on rated velocity assuming the ball screw operates within its rated velocity. Rated dynamic load, stroke length and weight data were collected from catalog data for 82 ball screws from one manufacturer (Nook Industries). Fig. 8 shows their weight as a function of rated load for two strokes.

An empirical equation for the lower bound of weight as a function of load and stroke was developed from the data

$$W_{\mathrm{bs}} = F_{\mathrm{bs}} \cdot \frac{180 + 3000 \cdot S_{\mathrm{bs}}}{10000} \quad (31)$$

where W_{bs} is ball screw weight, F_{bs} is ball screw rated dynamic load, and S_{bs} is ball screw stroke length. The equation is shown as the solid line in Fig. 8.

The transmission equations for the ball screw are

$$T_m = F_{\mathrm{bs}} \cdot \frac{n_{\mathrm{bs}}}{2\pi} \cdot \frac{1}{\eta_{\mathrm{bs}}} \quad (32)$$

$$\omega_m = \frac{V_{\mathrm{bs}}}{L_{\mathrm{bs}}} \quad (33)$$

where T_m is motor shaft torque (mN·m), F_{bs} is ball screw force (N), n_{bs} is the transmission ratio (mm/rev), η_{bs} is ball screw efficiency, ω_m is motor shaft velocity (rev/sec) and V_{bs} is ball screw linear velocity (mm/sec).

The ball screw efficiency was assumed to be 90%, which is typical for a high performance component. To simplify the electromechanical systems analysis, a fixed transmission ratio of 1 mm/rev was assumed for the ball screw.

C. Wire

The weight and efficiency of wire should be considered when analyzing electromechanical systems. The effects of the wire can be important when the wire carries large current over a signifcant distance, which would be the case when the battery is located some distance from the motor. High efficiency wire is large diameter, but heavy.

The voltage drop across a length of electrical wire is

$$\Delta U_{\mathrm{w}} = \frac{4K_{\mathrm{w}}}{\pi} \cdot \frac{P_w}{U_w} \cdot \frac{L_{\mathrm{w}}}{D_{\mathrm{w}}^2} \quad (34)$$

where K_{w} is wire specific resistance (Ohms· m), P_w is wire input power (W), U_w is wire input voltage (V), L_w is wire length (m) and D_w is wire diameter (m). Thus, wire efficiency is

$$\begin{aligned} \eta_{\mathrm{w}} &= \frac{U_w - \Delta U_{\mathrm{w}}}{U_w} \\ &= 1 - \frac{4K_{\mathrm{w}}}{\pi} \cdot \frac{P_w}{U_w^2} \cdot \frac{L_{\mathrm{w}}}{D_{\mathrm{w}}^2} \end{aligned} \quad (35)$$

High wire efficiency results in a large wire diameter and thus a large wire weight. In contrast, low wire efficiency means the wire must dissipate a considerable amount of thermal energy, which can melt the insulation or even the conductor. To prevent the system level weight optimization algorithm from suggesting either extreme, the wire efficiency was fixed at 99%, which is realistic for many systems.

Inverting (35), provides an equation for wire diameter

$$D_{\mathrm{w}} = \sqrt{\frac{4K_{\mathrm{w}}}{\pi} \cdot \frac{P_w}{U_w^2} \cdot \frac{L_{\mathrm{w}}}{(1 - E_{\mathrm{w}})}} \quad (36)$$

and the weight of the wire, without considering the insulation layer, is

$$W_{\mathrm{w}} = \frac{\pi}{4} \cdot D_{\mathrm{w}}^2 \cdot L_{\mathrm{w}} \cdot \rho_{\mathrm{w}} \cdot 1000 \quad (37)$$

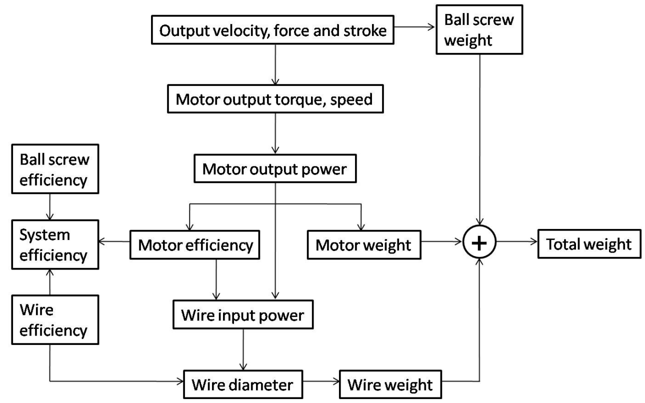

Fig. 9. Method for calculating the weight of an electromechanical system.

where W_{w} is wire weight, and ρ_{w} is the density of the wire material. The analysis only considered copper wire with density $8960\ \mathrm{kg/m^3}$ and specific resistance $17\ \mathrm{n\Omega m}$.

D. Electromechanical System Weight

Fig. 9 illustrates the approach for calculating the weight of the electromechanical solution.

The application requirements set the ball screw output velocity, force and stroke. The ball screw weight is then calculated using (31). The electric motor shaft power (in W) is calculated from

$$P_m = \frac{T_m}{1000} \cdot 2\pi \cdot \omega_m = \frac{F_{\mathrm{bs}} \cdot V_{\mathrm{bs}}}{\eta_{\mathrm{bs}}} \quad (38)$$

using (32) and (33). This determines the motor weight and efficiency according to (29) and (30). Next, the input power to the wire is determined from

$$P_w = P_m \cdot \frac{1}{\eta_{\mathrm{m}}} \cdot \frac{1}{\eta_{\mathrm{w}}} \quad (39)$$

and then the wire diameter and wire weight are calculated from (36) and (37). The system weight is the sum of the ball screw, motor and wire weights. The overall electromechanical system efficiency is

$$\eta_{\mathrm{esys}} = \eta_{\mathrm{bs}} \cdot \eta_{\mathrm{m}} \cdot \eta_{\mathrm{w}} \quad (40)$$

V. Method to Compare Hydraulic and Electromechanical Systems

With the ability to calculate system weight and efficiency for given application requirements for hydraulic and electromechanical realizations, it is possible to compare the two solutions to determine which is lighter. The method used for the comparisons was to: (1) Establish the design problem by specifying a system force and power (or force and velocity), and linear excursion. (2) Design an electromechanical solution using the empirical bounding equations as a stand-in for the best-available DC brushless motor and ball screw. (3) Calculate the efficiency of the resulting electromechanical system. (4) Design a comparable hydraulic system with the same force, power and stroke design requirements and the

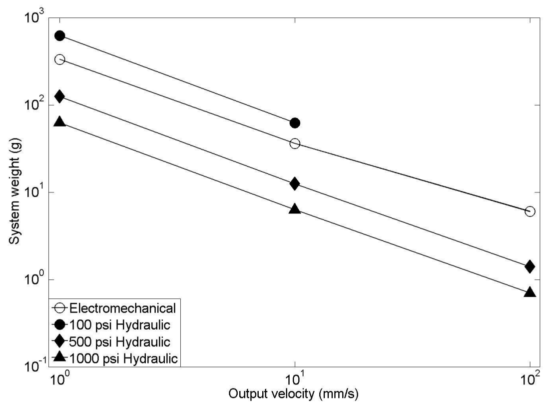

Fig. 10. Hydraulic and electromechanical system weight at several output velocities. Output power: 10 W, stroke: 0.05 m, transmission line length: 0.1 m. The 100 psi, 100 mm/s data point is missing because there is no low pressure, high speed hydraulic system that can match the efficiency of the equivalent electromechanical system.

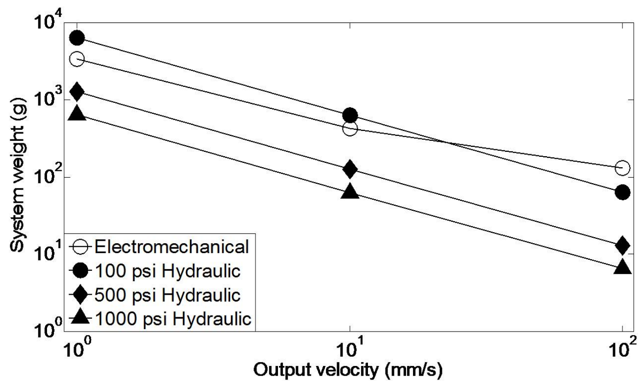

Fig. 11. Hydraulic and electromechanical system weight at several output velocities. Output power: 100 W, stroke: 0.05 m, transmission line length: 0.1 m.

same efficiency. (5) Calculate and compare the weights of the electromechanical and hydraulic solutions. A calculator application, based on the equations from the previous sections, was implemented in Matlab to facilitate the comparisons.

VI. Results

Examples of comparing solution weights are shown in the following figures. Figs. 10-15 show system weight for a mechanical output power of 100 W and 10 W for various configurations of velocity, stroke length and transmission line length. The nominal voltage for the motors in the database ranged from 6 to 48 V and for this analysis, 24 V was selected. Motor voltage has some, but not a significant effect on the electromechanical system weight. As the voltage decreases, the system weight will increase because the wire diameter must go up to accommodate the increase in current at the same efficiency.

One of the most significant factors that influences the weight of a hydraulic system is the nominal operating pressure. Fig. 16 shows the weights of hydraulic systems running at

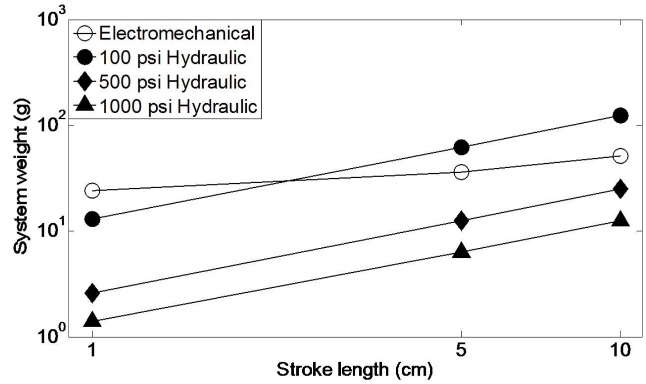

Fig. 12. Hydraulic and electromechanical system weight at several stroke lengths. Output power: 10 W, velocity: 0.01 m/s, transmission line length: 0.1 m.

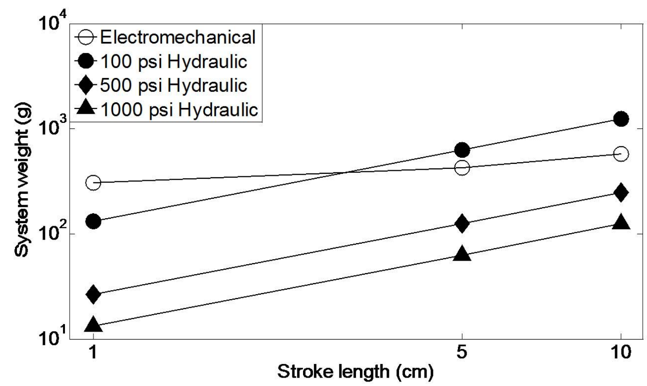

Fig. 13. Hydraulic and electromechanical system weight at several stroke lengths. Output power: 100 W, velocity: 0.01 m/s, transmission line length: 0.1 m.

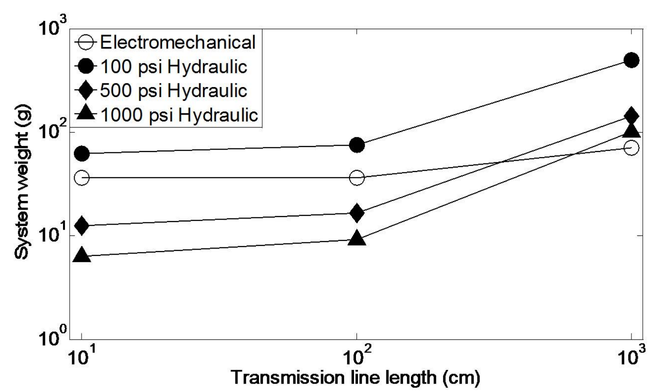

Fig. 14. Hydraulic and electromechanical system weight at several transmission line lengths. Output power: 10 W, stroke: 0.05 m, velocity: 0.01 m/s.

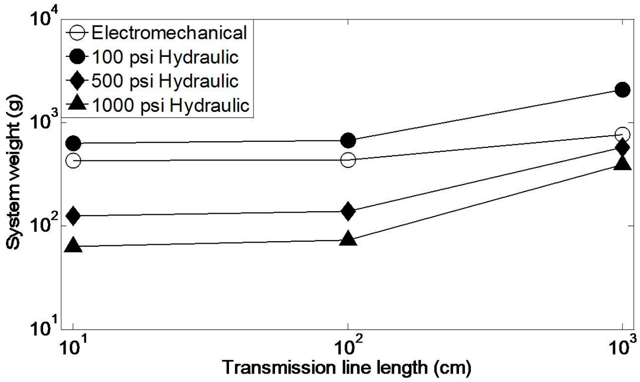

Fig. 15. Hydraulic and electromechanical system weight at several transmission line lengths. Output power: 100 W, stroke: 0.05 m, velocity: 0.01 m/s.

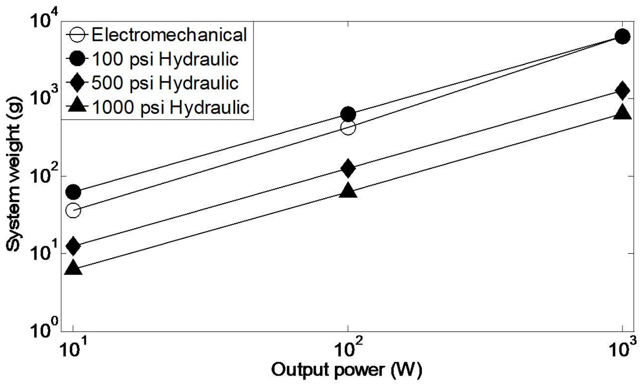

Fig. 16. Hydraulic and electromechanical system weight at several output powers. Stroke: 0.05 m, velocity: 0.01 m/s, transmission line length: 0.1 m.

several pressures compared to the weight of the equivalent electromechanical system for three output power conditions with an output velocity of 10 mm/s. The 100 psi hydraulic system will be heavier than the equivalent electromechanical system while the 500 psi and 1000 psi systems will be lighter.

Fig. 17 shows the operating pressure required for the hydraulic system to have the same weight as the equivalent electromechanical system for three output powers. Pressures higher than the line will result in a lighter hydraulic system and pressures below the line will result in a heavier hydraulic system.

VII. Discussion

The key result of this study, indicated in Figs. 16 and Fig. 17, is that for low power applications (< 100 W), a hydraulic solution will only be lighter than the equivalent electromechanical solution, if the hydraulics are run at high pressure. Fig. 16 shows that a hydraulic solution can be significantly lighter than an electromechanical solution if the pressure is very high. For example, a 100 W electromechanical system is predicted to weigh 428 g while a 100 W hydraulic system running at 1000 psi is predicted to weigh 63 g, about seven times lighter. While the exact numbers are system dependent (for example as the power source is placed further away, the drag in small hydraulic lines become significant),

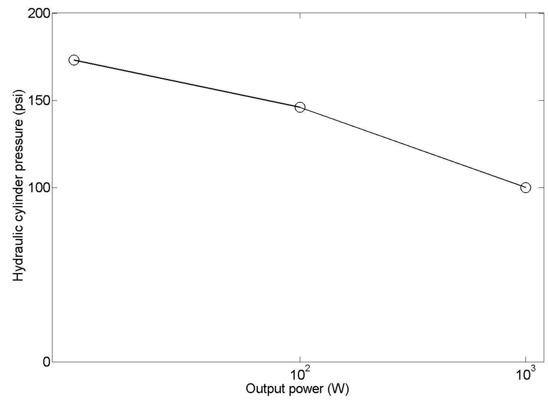

Fig. 17. Operating pressure required for the hydraulic system to be the same weight as the equivalent electromechanical system at several output powers. Stroke: 0.05 m, velocity: 0.01 m/s, transmission line length: 0.1 m.

the conclusion is clear: for tiny, light hydraulic systems the operating pressure should be high.

At higher power, the weight advantage of hydraulics is evident even at low pressure. In Fig. 16 a 1,000 W electromechanical system would weigh 6.3 kg and a 100 psi hydraulic system would also weigh 6.3 kg. Moving to 1,000 psi, the hydraulic system would weigh 0.63 Kg, ten times lighter. The weight difference increases in significance beyond 1,000 W, which is why hydraulics are the standard for high power applications such as excavators. What this study shows is that hydraulics remain favorable at low power, but only if the pressure is high. If the pressure is low, the electromechanical system will be lighter.

Because tiny high pressure hydraulic components are not available off the shelf, most small hydraulic systems are run at low pressures, often using pneumatic components that are small and light but generally limited to about 200 psi. An exception is the work by Love et al. [9] who prototyped tiny, high pressure hydraulics for use in prosthetic fingers. There is a clear need for innovation in the development of small components that operate at high pressures and can meet the weight predicted by Eqns. (16) and (24). In particular, the inset for Fig. 4 shows that at small scales there is room for improved cylinders that are lighter than what is commercially available.

The main limitation of the analysis is that it ignores the power supply and control means (see Fig. 1), which are significant components of the complete system. Analyzing the distal components of the system, which was done in this study, still provides guidance to the designer for two reasons. First, as mentioned previously, it is often the distally mounted components that are most weight sensitive and second, including the power supply and control would not change the main conclusion which is that tiny hydraulics must be run at high pressure to maintain their weight advantage over tiny electromechanical solutions.

Turning to the complete hydraulic system, hydraulic power supplies are typically large and heavy and traditional throttling valves are extremely inefficient. For truly lightweight, low power, mobile systems such as powered hand tools and powered orthotics, there is a need to develop compact sources of high pressure fluid using pumps driven by battery powered motors or driven by tiny, high power density internal combustion engines. Tiny cartridge piston pumps are available but have a modest efficiency of about 30%.

There is also a need for tiny, high pressure, low flow hydraulic control valves that operate in an efficient on-off switching mode. PWM drivers for electric motors are a well-developed, highly efficient control means that have no equivalent in fluid power. Low-pressure, low flow MEMS valves are common in micro-fluidics, but not suitable for transmitting power in the one to 100 W range.

Other problems with hydraulics that must be solved include leakage of oil into the environment, which calls for continuing research in low friction, leakless seals; cavitation of the fluid, which may be a significant problem for oil running through small passages at low pressure and high velocity; and creating designs that integrate structure, conduit, valving and cylinders to minimuze weight by eliminating fittings.

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Professor Chris Paredis and his students at the Georgia Instittute of Technology created the database of hydraulic cylinder properties. This research was supported by the Center for Compact and Efficient Fluid Power, a National Science Foundation Engineering Research Center, funded under cooperative agreement number EEC-0540834.

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