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Computation of Optimum Consumption of Energy for Anthropomorphic Robot Driven by Electric Motor

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15.1 Introduction

The optimization problem for robot engineering systems according to the criterion of minimum consumed work is one of the important and complicated problems in robotics. Difficulties of this problem are caused by multi-parametrization, large number of degrees of freedom of the system, high order and bulkiness of differential equations systems, non-smoothness functional of energy consumption, etc.

In spite of a large number of works, which deal with this problem, there still arise many important questions in design of systems with practically important types of drives. These demands stimulate new approaches and solutions. From the point of view of consumed energy, walking robots need more energy than wheeled platforms. Consequently, walking robots require larger electric motors and larger sources of energy. Most of walking robots are autonomous devices, in which duration of functioning is determined both by the capacity of the onboard power supply and the speed of consumption of energy. Therefore, search of theoretically achievable estimations of energy consumption from the point of view of choice of optimum laws of control and development of appropriate mathematical models and software are very urgent.

In the paper, the method of numerical parametrical optimization problem for biped walking is proposed. As an object of the research, the model of the inverted mathematical pendulum, to which the moment is applied by the electric motor, and the model of a multi-link planar walking robot (biped walking robot) consisting of a heavy trunk and a pair of weightless identical legs are considered. Notice, that with the help of this model in monographs [1,4], estimation of a single support phase of a biped walking robot movement was received, when only one leg is supporting on the surface, while the other leg is in phase of moving. In comparison to [1, 4], walking (gait) process is analyzed taking into account processes taking place in circuits of electric motors in the presented paper. The purpose of this work is to develop and improve methods of estimation of power consumption for walking robots and to

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create the appropriate software with simulation of computer visualization of the studied process of walking (gait). The paper is a continuation of [5].

For research of power consumption in robot walking, methods of theoretical mechanics, the theory of optimization, the theory of ordinary differential equations, numerical methods and simulation of computer visualization are used.

15.2 Equation of Motion for Inverted Pendulum

The electromechanical system consisting of the planar inverted pendulum and the direct current electric motor, which applies a moment to the pendulum through reducer (Fig. 15.1), is considered in the paper. It is the model of the moment, which is produced in the foot of the walking robot.

Point O is the point of support of the walking robot. The position of the pendulum is determined by the angle of turn φ, which is counted counter-clockwise from the fixed horizontal X axis of the coordinate system Oxy. Let the weight of pendulum be equal to m, the distance between point O and the center of weight be h, the moment of inertia of the pendulum with respect to the suspension axis be J_0, and the moment of inertia of the motor anchor be J_m. The effective moment of inertia is J. The values of J_0 and J_m are included in J $(J = J_0 + J_m n^2)$. Lagrange's function L and Rayleigh's function $\varPhi$ for the considered electromechanical system are set in the form:

$$L = \frac{1}{2}J\dot{\varphi}^2 + \frac{1}{2}L_0 i^2 + c_1 n \varphi i - mgh\sin\varphi, \qquad \varPhi = \frac{1}{2}\beta\dot{\varphi}^2 + \frac{1}{2}Ri^2. \tag{15.1}$$

Here L_0, R – generalized inductance and resistance of the rotor windings, i – current in the external circuit of the rotor, c_1 – factor of the electromechanical interaction, n – gear ratio of the reducer, β – factor of the viscous friction.

The equation of the pendulum movement with the direct current electric motor may be written in the form of Lagrange-Maxwell equation:

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} + \frac{\partial \varPhi}{\partial \dot{\varphi}} = 0, \\ &\frac{d}{dt}\left(\frac{\partial L}{\partial i}\right) + \frac{\partial \varPhi}{\partial i} = U. \end{aligned} \tag{15.2}$$

Here U – voltage, which is supplied to the motor. Substitution (15.1) into (15.2) leads to a system of two nonlinear differential equations as follows:

$$\begin{cases} J\ddot{\varphi} + \beta\dot{\varphi} + mgh\cos\varphi + c_1 n i = 0, \\ L_0\dfrac{di}{dt} + c_1 n\dot{\varphi} + Ri = U. \end{cases} \tag{15.3}$$

After separating the current i from the first part of equation (15.3) and substituting into second part, the equation leads to one nonlinear differential equation of the third order as follows:

$$\begin{aligned} JL_0\frac{d^3\varphi}{dt^3} + (JR + L_0\beta)\frac{d^2\varphi}{dt^2} + (c_1^2 n^2 + R\beta - mghL_0\sin\varphi)\frac{d\varphi}{dt} + \\ + mghR\cos\varphi = c_1 nU. \end{aligned} \tag{15.4}$$

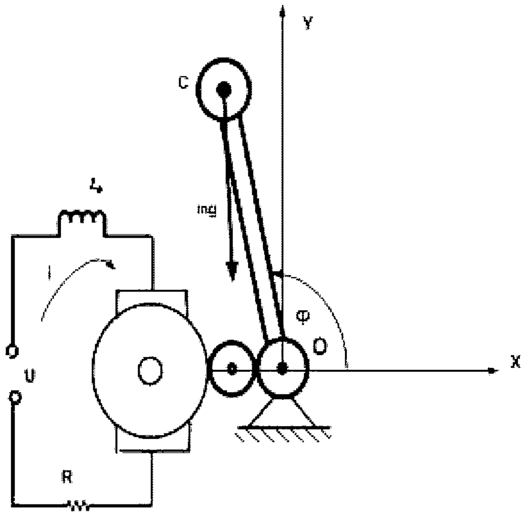

Fig. 15.1. Planar inverted pendulum

The length of the step is specified as L_s, the time of the step is t_k, and then we have two boundary-value conditions for the angle φ:

$$\varphi\big|_{t=0} = \varphi_0, \qquad \varphi\big|_{t=t_k} = \varphi_k. \tag{15.5}$$

For simplicity, the initial and final positions of the pendulum with respect to the vertical are supposed symmetric, so equations (15.5) become: $\varphi_k = \pi/2 - \arcsin\frac{L_s}{2h}$, $\varphi_0 = \pi/2 + \arcsin\frac{L_s}{2h}$.

For construction of periodic movement of the walking robot, we have to make equal the angular speed of the center of weight of the pendulum at the beginning and at the end of a single support phase:

$$\dot{\varphi}\big|_{t=0} = \dot{\varphi}\big|_{t=t_k}. \tag{15.6}$$

The programmed movement, which satisfies condition (15.5) is formulated in the form of the third-order Lagrange interpolation polynomial:

$$\begin{aligned}\varphi = \varphi_0 &\frac{(t_1-t)(t_2-t)(t_k-t)}{(t_1-t_0)(t_2-t_0)(t_k-t_0)} + \alpha_1\frac{(t-t_0)(t_2-t)(t_k-t)}{(t_1-t_0)(t_2-t_1)(t_k-t_1)} + \\ + \alpha_2&\frac{(t-t_0)(t-t_1)(t_k-t)}{(t_2-t_0)(t_2-t_1)(t_k-t_2)} + \varphi_k\frac{(t-t_0)(t-t_1)(t-t_2)}{(t_k-t_0)(t_k-t_1)(t_k-t_2)}.\end{aligned} \tag{15.7}$$

Here, α_1, α_2 are arbitrary constant values. They are functions of $\varphi(t)$ and lie between points t_1, t_2. Let us assume for simplicity $t_1 = t_k/3$, $t_2 = 2t_k/3$, $\varphi_0 = \pi/2 + \Delta$, $\varphi_k = \pi/2 - \Delta$, and $\Delta = \arcsin\frac{L_s}{2h}$. Taking into account boundary-value condition (15.5) we find that $\alpha_2 = \pi - \alpha_1$. So, for the programmed movement (15.7), we obtain:

$$\varphi(t,\alpha_1) = \frac{9(6\alpha_1 - 2\Delta - 3\pi)}{2t_k^3}t^3 - \frac{27(6\alpha_1 - 2\Delta - 3\pi)}{4t_k^2}t^2 + \\ + \frac{(54\alpha_1 - 26\Delta - 27\pi)}{4t_k}t + \frac{\pi}{2} + \Delta. \tag{15.8}$$

Certainly, it is meaningful to consider only those programmed movements, at which angle φ monotonously decreases, i.e. at $0 \leqslant t \leqslant t_k$ the angular speed of the pendulum is negative:

$$\omega(t,\alpha_1) = \frac{27(6\alpha_1 - 2\Delta - 3\pi)}{2t_k^3}t^2 - \frac{27(6\alpha_1 - 2\Delta - 3\pi)}{2t_k^2}t + \\ + \frac{(54\alpha_1 - 26\Delta - 27\pi)}{4t_k} < 0. \tag{15.9}$$

The inequality (15.9) is carried out for all $0 < t < t_k$, when parameter α_1 is inside the interval $\frac{\pi}{2} + \frac{1}{27}\Delta < \alpha_1 < \frac{\pi}{2} + \frac{13}{27}\Delta$. At $L_s = 0.8$ m, $h = 1.2$ m, $\Delta = \arcsin \frac{L_s}{2h} \approx 0.34 \approx 19.5^\circ$ and the inequality (15.9) becomes $1.584 < \alpha_1 < 1.734$.

The energy consumption for the considered model is determined with the help of the functional

$$W = \int_{t_0}^{t_k} U(t)i(t)dt. \tag{15.10}$$

The subintegral expression in (15.10) is determined with use of equation (15.3) and (15.4) and has the following form:

$$U(t)i(t) = \frac{\left(\begin{array}{c} \left(JL_0\dfrac{d^3\varphi}{dt^3} + (JR + L_0\beta)\dfrac{d^2\varphi}{dt^2} + \right. \\ +(c_1^2n^2 + R\beta - mghL_0\sin\varphi)\dfrac{d\varphi}{dt} + \\ \left. + \, mghR\cos\varphi\right)(J\ddot{\varphi} + \beta\dot{\varphi} + mgh\cos\varphi) \end{array}\right)}{(c_1^2n^2)}. \tag{15.11}$$

The energy consumption for the walking robot in one step, according to formulae (15.10) and (15.11) for the programmed movement (15.7) is a function of one parameter α_1:

$$W = W(\alpha_1). \tag{15.12}$$

Thus, the problem of energy optimization for movement of the inverted pendulum is reduced to the search of the minimum of the function (15.12). The result of calculation according to formula (15.12) is plotted in Fig. 15.2, and in this case the form of transients of optimum angle $\varphi = \varphi(t)$ and $\omega = \omega(t)$ is close to movement of free pendulum $J\ddot{\varphi} + mgh\cos\varphi = 0$.

In this case, the following numerical data [3,4] are used for calculating the energy consumption: $m = 75$kg, $J = 100.8$kg·m^2, $h = 1.2$m, $g = 9.8$m/sec^2, $L_s = 0.8$m, $L_0 = 0.03$H, $R = 0.8\ \Omega$, $c_1 = 0.05$H·m/volt, $n = 300$, $\beta = 150$kg·m^2/sec, time of step $t_k = 2/3$sec. The nodal points of the polynomial interpolation are $t_1 = 2/9$sec and $t_2 = 4/9$sec. In addition, the mean velocity of motion of the center of mass in one step is $V_s = 1.2$m/sec. With the given numerical data we have $\Delta = 0.34 \approx 19.5^\circ$, and the mean angular velocity of the pendulum is $\omega(t,\alpha_1) \approx -1.02$/sec.

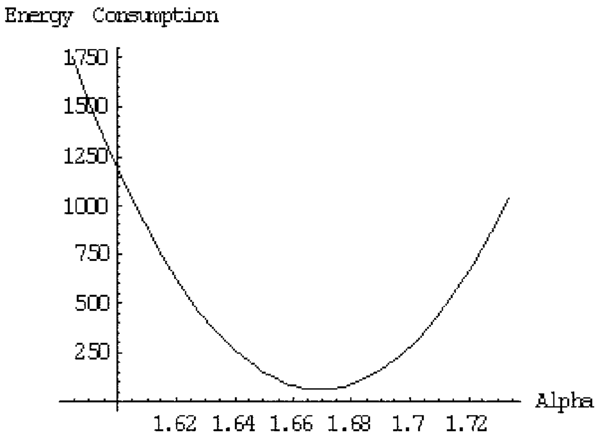

Fig. 15.2. Dependence of energy consumption on α

15.3 Exact Solution for Variational Problem

In this case, the transient in the circuit of the electric motor is neglected and it is assumed that in (15.4) $L_0 = 0$. Then for the case of a small deviation of the pendulum from the vertical, the problem minimization of the scalar energy criterion becomes

$$W = \int_0^1 F(u(\tau), x'(\tau))d\tau, \qquad F = \frac{k}{2}u^2 - bu\frac{dx}{d\tau}. \tag{15.13}$$

The variables of the functional (15.13) satisfy restrictions in the form of a differential equation:

$$\frac{d^2x}{d\tau^2} + \beta_1\frac{dx}{d\tau} - \Omega^2 x = u, \tag{15.14}$$

where

$$x = \varphi - \pi/2, \qquad \beta_1 = \frac{\beta t_k}{J} + \frac{c_1^2 n^2 t_k}{JR}, \qquad \Omega^2 = \frac{mght_k^2}{J},$$
$$u = \frac{c_1 n t_k^2}{JR}U, \qquad k = \frac{2J^2R}{c_1^2 n^2 t_k^4}, \qquad b = \frac{J}{t_k^3}.$$

For searching the optimal control $u(\tau)$, which minimizes functional (15.13), a method of classical calculus of variation of Euler-Lagrange [2] is used. As a preliminary, the equation (15.14) is written down in the normal form of Cauchy

$$x' = y, \qquad y' = \Omega^2 x - \beta_1 y + u, \tag{15.15}$$

where the accent sign denotes the derivative with respect to τ.

Let's write down the Hamilton function and then the system equations for the conjugated variable impulses

$$H = p_x y + p_y(\Omega^2 x - \beta_1 y + u) + \frac{k}{2}u^2 - buy, \tag{15.16}$$

$$p_x' = \frac{\partial H}{\partial y}, \qquad p_y' = -\frac{\partial H}{\partial x}. \tag{15.17}$$

From the minimum condition $\frac{\partial H}{\partial u} = 0$ of Hamilton function H

$$u = \frac{1}{k}(by - p_y). \tag{15.18}$$

Substituting (15.18) into (15.15) and (15.17) we come to the following system of the fourth-order linear differential equations:

$$\mathbf{X}' = \mathbf{AX}, \qquad \mathbf{X} = \begin{bmatrix} x & y & p_x & p_y \end{bmatrix}^T. \tag{15.19}$$

Here T – transposition sign,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \Omega^2 & \beta_{22} & 0 & -1/k \\ 0 & 0 & 0 & -\Omega^2 \\ 0 & b^2/k & -1 & -\beta_{22} \end{bmatrix}.$$

The eigenvalues of matrix $\mathbf{A}$ are determined after solving the following biquadratic equations:

$$\lambda^4 + \left(\frac{b^2}{k^2} - \beta_{22}^2 - 2\Omega^2\right)\lambda^2 + \Omega^4 = (\lambda^2 - \lambda_1^2)(\lambda^2 - \lambda_2^2) = 0. \tag{15.20}$$

The solution of Eq. (15.19) is

$$\mathbf{F} = \mathbf{S}.\exp(\Lambda\tau).\mathbf{S}^{-1} \tag{15.21}$$

where

$$\exp(\Lambda\tau) = \begin{bmatrix} e^{\lambda_1\tau} & 0 & 0 & 0 \\ 0 & e^{-\lambda_1\tau} & 0 & 0 \\ 0 & 0 & e^{\lambda_2\tau} & 0 \\ 0 & 0 & 0 & e^{-\lambda_2\tau} \end{bmatrix},$$

matrix $\mathbf{S}$ is formed from eigenvectors matrix $\mathbf{A}$, $\mathbf{S}^{-1}$ – inverse matrix of $\mathbf{S}$. Eigenvectors $\boldsymbol{s}_k$ are determined by formulas:

$$\boldsymbol{s}_1 = \begin{bmatrix} \frac{-1}{k\delta_1} & \frac{-\lambda_1}{k\delta_1} & \frac{-\Omega^2}{\lambda_1} & 1 \end{bmatrix}^T, \qquad \boldsymbol{s}_2 = \begin{bmatrix} \frac{-1}{k\delta_2} & \frac{\lambda_1}{k\delta_2} & \frac{\Omega^2}{\lambda_1} & 1 \end{bmatrix}^T,$$

$$\boldsymbol{s}_3 = \begin{bmatrix} \frac{-1}{k\delta_3} & \frac{-\lambda_2}{k\delta_3} & \frac{-\Omega^2}{\lambda_2} & 1 \end{bmatrix}^T, \qquad \boldsymbol{s}_4 = \begin{bmatrix} \frac{-1}{k\delta_4} & \frac{\lambda_2}{k\delta_4} & \frac{\Omega^2}{\lambda_2} & 1 \end{bmatrix}^T.$$

Here $\delta_1 = \lambda_1^2 - \lambda_1\beta_{22} - \Omega^2$, $\delta_2 = \lambda_1^2 + \lambda_1\beta_{22} - \Omega^2$, $\delta_3 = \lambda_2^2 - \lambda_2\beta_{22} - \Omega^2$, $\delta_4 = \lambda_2^2 + \lambda_2\beta_{22} - \Omega^2$.

The fundamental matrix (15.21) allows to write down the solution of Eq. (15.19) for the moment of time $\tau = 1$:

$$\mathbf{X}(1) = \mathbf{S}.\exp(\boldsymbol{\Lambda}).\mathbf{S}^{-1}\mathbf{X}(0). \tag{15.22}$$

Here $\mathbf{X}(0) = \begin{bmatrix}\Delta & \omega_0 & p_x^0 & p_y^0\end{bmatrix}^T$, $\mathbf{X}(1) = \begin{bmatrix}-\Delta & \omega_0 & p_x(1) & p_y(1)\end{bmatrix}^T$ and, hence, the first term of Eq. (15.22) represents a system of two linear algebraic equations for initial values of impulses p_x^0, p_y^0. The solution of the indicated equations determines completely the behavior of the trajectories of system (15.19), and substituting formula (15.19) into (15.18) gives optimal control of energy for the electric motor.

15.4 Equation of Motion for Multi-link of the Walking Robot

The proposed method of parametric optimization of variational problem for searching for the minimum of a function of several variables in the first section is used in optimizing the problem of walking for an electromechanical biped walking robot. The model of the planar anthropomorphic walking robot consists of a heavy trunk and a pair of weightless identical legs is considered. The movement of the walking robot is provided with direct current electric motors, which create moments in the hinges of the robot.

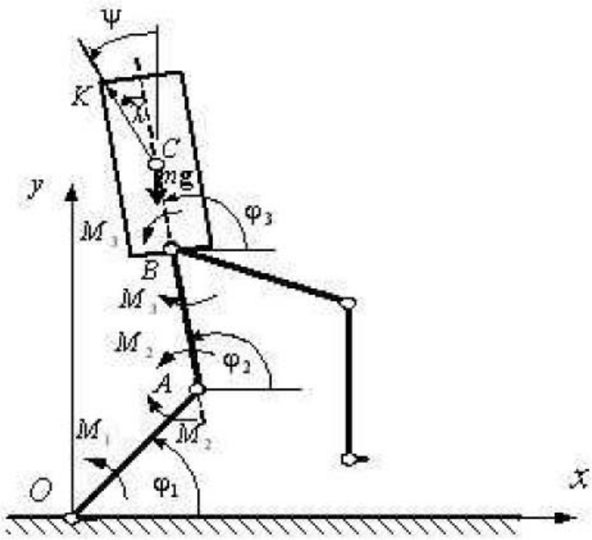

Fig. 15.3. Model of a five-link planar walking robot

In the given paper the walking process is analyzed taking into account the processes that run in the circuit of the electric motors. In a single support phase the considered electromechanical system of the walking robot has six degrees of freedom: three angles $\varphi = \begin{bmatrix}\varphi_1 & \varphi_2 & \varphi_3\end{bmatrix}^T$, which specify the position of links OA, OB, BC and three currents $\mathbf{i} = \begin{bmatrix}i_1 & i_2 & i_3\end{bmatrix}^T$, which flow in the circuits input of direct current electric motors, which create the moments in the hinges O, A, B (Fig. 15.3).

The equations of motion for the proposed electromechanical system in a single support phase represent a system of second order nonlinear differential equations for a vector of angular variables and a vector of currents

$$\begin{aligned} &\mathbf{A}\ddot{\varphi} + \mathbf{B}\dot{\varphi}^2 - \mathbf{P} = \mathbf{K}.\mathbf{M}, \\ &\mathbf{L}\frac{d\mathbf{i}}{dt} + R\mathbf{i} + \mathbf{C}\dot{\varphi} = \mathbf{U}, \end{aligned} \tag{15.23}$$

where

$$\mathbf{A} = m \begin{bmatrix} -l_1 \sin\varphi_1 & -l_2 \sin\varphi_2 & -l_3 \sin\varphi_3 \\ l_1 \cos\varphi_1 & l_2 \cos\varphi_2 & l_3 \cos\varphi_3 \\ 0 & 0 & \rho^2 \end{bmatrix},$$

$$\mathbf{B} = m \begin{bmatrix} -l_1 \cos\varphi_1 & -l_2 \cos\varphi_2 & -l_3 \cos\varphi_3 \\ l_1 \sin\varphi_1 & l_2 \sin\varphi_2 & l_3 \sin\varphi_3 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{K} = \frac{1}{s_{21}} \begin{bmatrix} l_2 \cos\varphi_2 & -l_1 \cos\varphi_1 - l_2 \cos\varphi_2 & l_1 \cos\varphi_1 \\ l_2 \sin\varphi_2 & -l_1 \sin\varphi_1 - l_2 \sin\varphi_2 & l_1 \sin\varphi_1 \\ s_{32} & -s_{32} - s_{31} & s_{31} + s_{21} \end{bmatrix},$$

$$s_{21} = l_1 l_2 \sin(\varphi_2 - \varphi_1), \quad s_{32} = l_2 l_3 \sin(\varphi_3 - \varphi_2), \quad s_{31} = l_1 l_3 \sin(\varphi_3 - \varphi_1),$$

$$\ddot{\varphi} = \begin{bmatrix} \ddot{\varphi}_1 & \ddot{\varphi}_2 & \ddot{\varphi}_3 \end{bmatrix}^T, \qquad \dot{\varphi}^2 = \begin{bmatrix} \dot{\varphi}_1^2 & \dot{\varphi}_2^2 & \dot{\varphi}_3^2 \end{bmatrix}^T,$$

$$\mathbf{P} = \begin{bmatrix} 0 & mg & 0 \end{bmatrix}^T, \qquad \mathbf{M} = \begin{bmatrix} M_1 & M_2 & M_3 \end{bmatrix}^T.$$

The movement law for each link of walking robot $\varphi_1 = \varphi_1(t)$, $\varphi_2 = \varphi_2(t)$, $\varphi_3 = \varphi_3(t)$ as well as in case of the inverted pendulum is set in the form of polynomial Lagrange interpolation as in (15.7).

In determining the programmed values of the motor moments at which the programmed movement is realized, it is assumed that in this movement there is no straightening of the legs of the walking robot, i.e. $\varphi_1(t) \neq \varphi_2(t)$. Then matrix $\mathbf{K}$ represents a limited function of time, and from Eq. (15.23) it is possible to determine the controlling moments for electric motors as follows

$$\mathbf{M} = \mathbf{K}^{-1} \left(\mathbf{A}\ddot{\varphi} + \mathbf{B}\dot{\varphi}^2 - \mathbf{P} \right). \tag{15.24}$$

In this case after substituting the programmed movement into (15.24) it is necessary to check up realization of the conditions

$$|\mathbf{M}_k| \leqslant \mathbf{M}_k^\star, \tag{15.25}$$

where $\mathbf{M}_k^\star$ – maximal value of the moments, which are produced by the electric drives of the walking robot.

The functional, which determines consumption of energy

$$W = \int_{t_0}^{t_k} \mathbf{U}^T \mathbf{i} dt \tag{15.26}$$

is a function of a finite number of variables – angles of rotation of the links of the walking robot at the nodal points.

15.5 Conclusions

1. Computer simulation for movement of the considered walking robot shows that the offered technique allows to carry out a purposeful choice of design parameters of the walking robot, which use electric drives and programmed movement at walking.
2. Results of numerical experiments with respect to the proposed schema demonstrate the possibility of an essential reduction of energy consumption by the walking robot.

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