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A Method for Identification of Electrically Stimulated Muscle

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Abstract— **We present a model structure and a method for identifying the dynamics of electrically stimulated muscle. The model structure is sufficiently rich to describe a wide set of muscle behavior. It consists of (i) an input static nonlinearity representing the muscle's recruitment properties, (ii) a linear dynamical system representing the contraction dynamics, (iii) an output static nonlinearity representing generalized force-length and force-velocity relationships, and (iv) prefilters for the mechanical input that capture impedance and history dependence properties of the muscle. It is assumed that each of the subsystems is linearly parameterized. We present parameter estimation methods, and verify via simulation successful convergence of the estimates to their true values with small variances.**

I. INTRODUCTION

Identification of the dynamics of electrically stimulated muscle is essential for developing stable adaptive controllers for functional electrical stimulation (FES). In the FES literature, there has been substantial research outlining methods for identifying muscle response, both *in vivo* [1], [2], [3], [4] and *ex vivo* [5], [6], [7]. While identifying muscle response, in its most general form, is a challenging problem, tractability is typically achieved in two ways. The first is to confine the muscle response to a narrow range of operating conditions. An example of this approach is to focus on isometric contractions as in [5] and [7]. The second way of simplification is to assume a predetermined structure that makes the identification process easier. For example, in [3], [6], a multiplicative structure for the force-length, force-velocity and activation was assumed. In [8] a neural network model estimated the recruitment nonlinearity, coupled with third order linear dynamics.

Furthermore, in the muscle physiology literature, there has been significant work characterizing a muscle's response, in the form of force-length and force-velocity curves. See [9], [10] for a survey. Emphasis on such research has been on characterizing muscle under full recruitment. Such conditions are not biologically realistic, and hence models based on them may not be fully justified from a FES point of view. Additionally, the experimental protocols utilized therein typically estimated the parameters in sequence. This requires time consuming experimentation that induce rapid fatigue.

In this paper we propose a model structure and a corresponding identification procedure. The model structure is rich enough to cover a wide set of muscle behaviors. At its core is a Hammerstein model, which has been successful in the identification of isometric contraction. We augment it with an additional nonlinearity that models a muscle's force-length and force-velocity characteristics, and utilize general functional forms that impose little restrictions on those characteristics. All nonlinearities are approximated using Gaussian radial basis functions, and therefore do not assume particular structures. Finally, we include prefilters that capture the muscles impedance and history dependence properties. This flexible structure allows for wider applicability under more generalized conditions. We also provide an identification scheme capable of estimating the model parameters well. We show via simulation good convergence to the true values with small variances.

II. MUSCLE MODEL STRUCTURE AND PARAMETERS

A. Model Structure

The model structure proposed is shown in Figure 1. The model inputs are electrical stimulus $u(t)$ and the muscle's mechanical state (its strain $x(t)$ and strain-rate $\dot{x}(t)$). The output $y(t)$ is the contractile force generated by the muscle due to both its active and passive components. Note that the internal variables $v(t)$, $z(t)$, $x_f(t)$ and $\dot{x}_f(t)$ are not accessible experimentally. The different blocks of the model are justified as follows:

1) The input nonlinearity $f(u)$: The input to the system u can represent any parameter used to modulate the muscle's activation. This may include pulse amplitude, pulse frequency, or pulse-width (see for examples [11], [12], [13], [14], [6], [7]). The nonlinearity $f(u)$ typically assumes a sigmoid shape regardless of the modulation parameters employed. For fatigue purposes, pulse-width is typically preferred as the modulation parameter.

2) The linear dynamics block $G(q)$ (where q is the time shift operator): Models the dynamics of calcium diffusion to and from the sarcoplasmic reticula, giving rise to contraction dynamics. Many researchers reported, based on experimental data, that second or third order models are sufficiently accurate [6], [7], [8], [3]. We therefore model this as an auto-regressive process with exogenous input (ARX).

3) The prefilters $L_x(q)$ and $L_v(q)$: Act upon the mechanical input to the muscle, and are used to capture two sets of muscle behaviors. The first is the set is the muscle response under quick release experiments [10], [9]. Under constant excitation, muscles exhibit the behavior of similar to that of a series spring with a parallel damper. Such behaviors may be captured by appropriate transfer functions. The second set of behaviors deals with the long-term history

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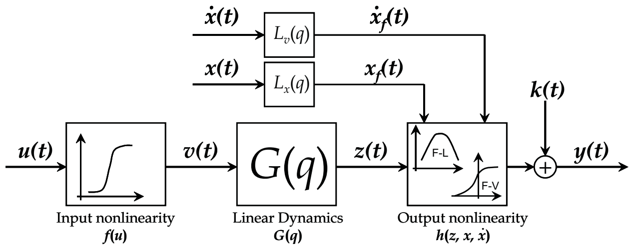


Fig. 1. Proposed muscle model structure. The input nonlinearity $f(u)$ represents recruitment characteristics. The LTI block $G(q)$ models contraction dynamics. The output nonlinearity $h(z,x,\dot{x})$ models force-length and force-velocity curves. The systems L_x and L_v prefilter the mechanical input to the muscle (both its strain, x, and strain rate, $\dot{x}$). This captures the muscle's impedance, and history dependence on the mechanical response. Sensor noise $k(t)$ is inserted additively.

dependence properties of muscles (see [15] for a survey and the references therein). It is well known that the muscle exhibit force enhancement due to prior stretching and force attenuation due to prior shortening. Such effects may be modeled using appropriate states in the prefilters.

4) The output nonlinearity $h(z,x_f,\dot{x}_f)$: Represents the relationship between the force produced and the mechanical state of the muscle (its strain and strain rate). Appropriate cross-sections of the function represent the force-length (strain) and force-velocity (strain rate) characteristics of the whole muscle, which modulate the muscle response. While the muscle physiology literature is rich in such characterizations at either constant length or constant velocity, very little data exists on generalized mechanical conditions.

Note that while the model structure has a nonlinear-linear-nonlinear cascade similar to that of a Hammerstein-Weiner cascade, the output y is assumed to be linear in z. It is simply modulated by the filtered mechanical input to the muscle as defined by x_f and $\dot{x_f}$. Therefore the output nonlinearity takes the form of $h(z,x_f,\dot{x}_f)=\tilde{h}(x_f,\dot{x}_f)\cdot z$. This assumption simplifies the identification scheme considerably since the dimensionality of the function $\tilde{h}(x_f,\dot{x}_f)$ is reduced.

Thus, with slight abuse of notation, the model becomes:

$$\begin{array}{lllllll} v(t) & = & f(u(t)) & ; & z(t) & = & G(q)v(t) \\ x_f(t) & = & L_x(q)x(t) & ; & \dot{x}_f(t) & = & L_v(q)\dot{x}(t) \\ y(t) & = & \tilde{h}(x_f(t),\dot{x}_f(t))\cdot z(t) & & & & \end{array}$$

Note that $f(\cdot)$ and $G(\cdot)$ will be unique up to their relative gains. Therefore, to ensure strict identifiability, we fix the gain of $G(\cdot)$ to unity. Similarly, we fix the gains of the function $L_x(\cdot)$ and $L_v(\cdot)$ to one, while allowing $\tilde{h}(\cdot)$ to set amplitude levels.

B. Model Parameterization

Each of the blocks of Figure 1 is assumed to be linearly parameterized as follows:

- The input nonlinearity $f(u)$ is expanded as a linear combination of basis functions. Many candidate functions are possible. For example, Tchebyshev polynomial basis functions are attractive since they maintain orthogonality with respect to bounded input ranges [16]. However, Gaussian radial basis functions are chosen here due to the generality of their approximation capability and due to their locality. Therefore,

$$f(u)=\sum_{i=1}^{n_f} Y_i(u)\theta_{f_i}=\varphi_f\theta_f \qquad (1)$$

where $Y_i(a)=\exp\left(-\frac{\|a-\zeta_i\|^2}{\sigma_i}\right)$ and, ζ_i and σ_i are the centers and spreads of the local basis functions respectively, n_f is the number of basis functions and φ_f is the regressor (clearly defined from above). For simplicity we fix the grid centers ζ_i and spreads σ_i [17], although adaptive methods may be incorporated. Note that other functions that mimic the sigmoid shape are generally not linear in their parameters, thereby making parameter estimation cumbersome. Therefore, under this representation, θ_f parameterizes the function $f(\cdot)$.

- The ARX model of $G(q)$ is given as

$$\begin{aligned} z(t) &= \frac{B_G(q)}{A_G(q)}v(t) \\ &= [v(t),\cdots,v(t-n_b),z(t-1),\cdots,z(t-n_a)]\theta_G \\ &= \varphi_G^T(u(t),\theta_f)\theta_G \end{aligned} \qquad (2)$$

where $\varphi_G(t)$ is the linear regression vector.

- The output nonlinearity $\tilde{h}(x,\dot{x})$ is also defined in terms of Gaussian basis functions akin to $f(u)$. Therefore,

$$h(u) = \sum_{i=1}^{n_h} Y_i(x)\theta_{h_i}=\varphi_h\theta_h \qquad (3)$$

where we use a two-dimensional form of $Y_i(\cdot)$.

- The prefilters are characterized in a manner similar to $G(q)$. Therefore we write:

$$x_f(t) = \frac{B_{L_x}(q)}{A_{L_x}(q)}x(t)=\varphi_{L_x}^T(t)\theta_{L_v} \qquad (4)$$

$$\dot{x}_f(t) = \frac{B_{L_v}(q)}{A_{L_v}(q)}\dot{x}(t)=\varphi_{L_v}^T(t)\theta_{L_x} \qquad (5)$$

with $\varphi_{L_x}(t)$ and $\varphi_{L_v}(t)$ being the appropriate regressors.

In summary, the dynamics of the muscle is parameterized by the vector $\Theta=[\theta_f^T \quad \theta_G^T \quad \theta_h^T \quad \theta_{L_x}^T \quad \theta_{L_v}^T]^T$.

III. MODEL IDENTIFICATION

We propose a two stage procedure to identify the parameters of the structure of Figure 1. The first stage allows for identifying $f(\cdot)$ and $G(\cdot)$ simultaneously, while in the second stage, the parameters of $h(\cdot)$, $L_x(\cdot)$ and $L_v(\cdot)$ are estimated.

A. Stage I

In the first stage, the muscle is maintained under isometric conditions. Since the mechanical state $[x,\dot{x}]$ will not change, the function $\tilde{h}(\cdot)$ reduces to a constant scalar value $\tilde{h}_{iso}$. Without loss of generality, we can set $\tilde{h}_{iso}=1$. The system is stimulated via white-noise stimulation, while the output force of the system is recorded. Therefore, as per our model structure, the output y will be equal to z, i.e. $y(t)=z(t)$. Thus the system reduces to a basic Hammerstein structure, and the nonlinearity $\tilde{h}(\cdot)$ will not take effect.

Parameter estimation is cast in a minimum prediction error framework. Defining the quadratic cost function

$$J_N(\Theta) = \frac{1}{2}\sum_{t=1}^{N} \|\hat{y}(t,\Theta) - y(t)\|^2$$

Our objective is to pick the parameter estimates characterized by $\hat{\Theta} \in \arg\min J_N(\Theta)$.

Under our representation, all components of the system are individually linearly parameterized. However the overall system is not linearly parameterized due to the cross products between θ_f and θ_G. This implies that linear least squares cannot be applied directly.

A key observation is that the system is separably linear in the parameters, i.e. fixing θ_f provides a linear parameterization in θ_G and vice-versa. This observation suggests a recursive procedure for parameter estimation. This procedure was first proposed in [18]:

$$\hat{\theta}_G^{k+1} = \arg\min_{\theta_G} J_N(\hat{\theta}_f^k, \theta_G) = \left(M^T M\right)^{-1} M^T y \quad (6)$$

where $M = \varphi_G(u(t), \hat{\theta}_f^k)$. Similarly, we may write (in an abbreviated manner):

$$\hat{\theta}_f^{k+1} = \arg\min_{\theta_f} J_N(\theta_f, \hat{\theta}_G^{k+1}) \quad (7)$$

The minimization here is straightforward since the problem is separably linear in the parameters. Furthermore, recursive algorithms [19] may be applied.

B. Stage II

Now that the first two blocks for the model have been identified, stage two hinges on simulating the "isometric" subsystem defined by the parameters $\hat{\theta}_f$ and $\hat{\theta}_G$ to obtain an estimate $\hat{z}(t)$. This estimate is the used to obtain a standard least-squares fit of $\tilde{h}(\cdot)$. This is a two dimensional fit of the muscle's mechanical state $[x, \dot{x}]$ and the force produced. Similar to Stage I, the parameters to be estimated in Stage II will cross multiply, and a similar recursive procedure is employed:

$$\hat{\theta}_h^{k+1} = \arg\min_{\theta_h} J_N(\hat{\theta}_f, \hat{\theta}_G, \hat{\theta}_{L_x}^k, \hat{\theta}_{L_v}^k, \theta_h) \quad (8)$$

$$\hat{\theta}_{L_x}^{k+1} = \arg\min_{\theta_{L_x}} J_N(\hat{\theta}_f, \hat{\theta}_G, \theta_{L_x}, \hat{\theta}_{L_v}^k, \hat{\theta}_h^{k+1}) \quad (9)$$

$$\hat{\theta}_{L_v}^{k+1} = \arg\min_{\theta_{L_v}} J_N(\hat{\theta}_f, \hat{\theta}_G, \hat{\theta}_{L_x}^{k+1}, \theta_{L_v}, \hat{\theta}_h^{k+1}) \quad (10)$$

All the of the steps in this recursive procedure are linear fits.

C. Input signal selection

To fulfill the assumptions of [20], we use random noise input to excite the system mechanically and electrically (i.e. for $x(t)$ and $u(t)$). This is in contrast to using pseudo-random binary sequences (PRBS) which is commonly used in muscle identification. PRBS excitation is not informative enough identify the input nonlinearities [19]. Intuitively the function $f(u)$ will be sampled only at two points, and therefore cannot be reconstructed.

IV. Simulation Results

Simulation results demonstrating the method are shown in Figure 2. The assumed "true" system in the simulation was defined as follows:

- The true input nonlinearity $f(u)$ was a sigmoid function (shown in Figure 2(a)), with magnitude of 1 and exponential constant of 15.
- The true LTI system $G(q)$ was a second order system with two poles at 20 Hz.
- The true output nonlinearity is shown in Figure 2(b). It is constructed by taking the product of the force-length and force-velocity curves in Figures 2(e) and 2(f) respectively. The force-length curve is described by linear slopes smoothly merged at the plateau. The force-velocity curve is described by a Hill's relationship [10] in the contractile region, and exponential curve in the stretching region.
- For simplicity, the prefilters $L_x(q)$ and $L_v(q)$ were set to unity.

In the simulation, the input noise $k(t)$ was normally distributed with standard deviation $\lambda = 0.1$. The function $f(\cdot)$ was approximated by an equally spaced grid of 8×1 centers, whereas the grid for $\tilde{h}(\cdot)$ was 10×10. All spreads σ_i were set to 0.1. The initial conditions for all parameter estimates were set to one. The number iterations in Equations [6 through 10] was set to 10 iterations each.

Figure 2 shows plots of the assumed true system, as well as the identification results for 10 independent trials. The plots show that the identification procedure estimates the individual block parameters well. The plots of the estimates are narrowly clustered around the true values, indicating that the parameters are estimated with small covariance.

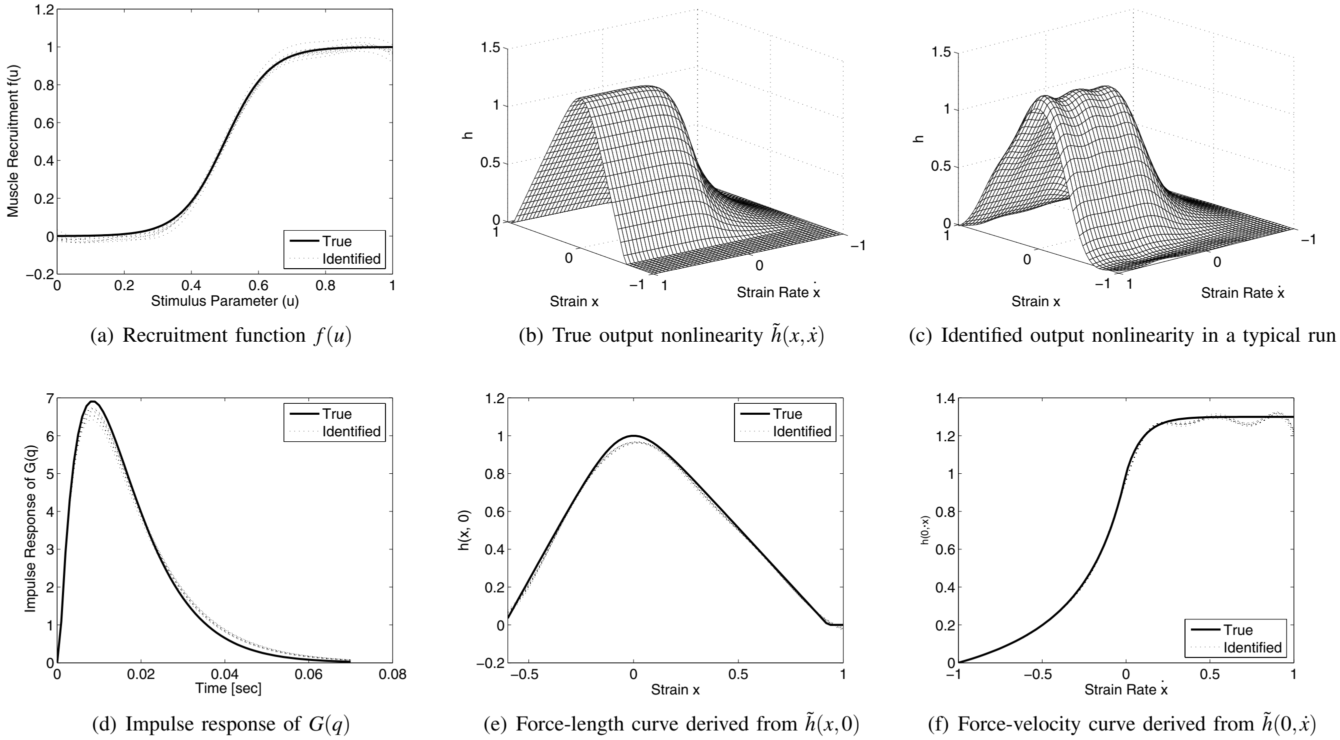
V. Discussion and Future Work

This paper provides proof-of-concept for an identification procedure to characterize the dynamics of electrically stimulated muscle. The method can characterize the different subsystems of the muscle structure proposed individually.

Future work includes asymptotic covariance analysis to formally quantify the effects of noise on the parameter estimates. The asymptotic covariance is intimately related to the sensitivity of the estimates with respect to the parameters and the noise variance [19]. Such analysis is necessary to obtain objective requirements on the length of data needed to achieve appropriate confidence levels of the parameter estimates.

Future work also includes determining be best grid parameters for the radial basis approximation. Figures 2(f) and 2(b) show ripple effects due to the Gaussian approximation, especially at the flat end of the plateau. Such effects may be attenuated by using wider spreads, but potentially at the expense of not capturing sharper features in the curves.

Ultimately, experimental results will determine the validity method. Experiments will require testing platforms that are capable of controlling the mechanical state (so as to induce the necessary perturbations of Stage II), while providing



(a) Recruitment function $f(u)$

(b) True output nonlinearity $\tilde{h}(x,\dot{x})$

(c) Identified output nonlinearity in a typical run

(d) Impulse response of $G(q)$

(e) Force-length curve derived from $\tilde{h}(x,0)$

(f) Force-velocity curve derived from $\tilde{h}(0,\dot{x})$

Fig. 2. Comparison of the true model (thick solid lines) and identification runs (fine dotted lines). The noise standard deviation was set at 0.1. Multiple runs of the estimation process give an indication of the covariance of the estimates. The curves of subfigures (e) and (f) are essentially cross-sectional cuts of Figures (b) and (c).

necessary random electrical stimulation. This can be accomplished experimentally as described in [21].

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