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Improvement of balancing accuracy of robotic systems: Application to leg orthosis for rehabilitation devices

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Abstract

The balancing of robotic systems is an important issue because it allows significant reduction of torques. However the literature review shows that the gravity balancing of robotic systems is carried out by weightless springs. For many balancing schemes it is the source of errors.

This paper deals with an analytically tractable solution for the gravity balancing considering the spring mass. For this purpose, the relationship between the stiffness coefficient of the spring and its mass is provided. Then this relationship is introduced into the balancing equation and spring elastic force is determined taking into account its mass. For zero length springs, the stiffness coefficient of the springs is determined from a quadratic equation and for non-zero length springs from a cubic equation. In this way, an exact balancing of gravitational forces is achieved, which allows improving the balancing accuracy of robotic systems.

The efficiency of the suggested approach is illustrated by numerical examples. An application to the balancing of the leg orthosis for robotic rehabilitation is also presented.

Keywords: Gravity balancing; Spring balancer; Robotic system; Rehabilitation devise

1. Introduction

Balancing of mechanisms is a well-known research field, which finds practical applications in high-speed machines, machine-tool industry, robotics and in many other branches of industry. Despite its ancient history [1–3], mechanism balancing theory continues to develop and new approaches and solutions are constantly being reported. A new field of its applications is the robotic rehabilitation [4–7] and the parallel mechanisms [8–13].

The aim of present study is the balancing of gravitational forces of robotic systems for minimization of input torques. For this purpose different approaches and solutions have been developed and documented.

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It can be carried out a systematization of these solutions by balancing means: counterweight, spring, pneumatic or hydraulic cylinder, etc. Taking into account that the suggested approaches may be applied in the rehabilitation devices, we limit our literature review only by spring balancers. Previous work on the design of spring balancers might be arranged in the following groups (Table 1):

A. Balancing by springs jointed directly with links (schemes A1–A6) [9,14–20].
B. Balancing by using a cable and pulley arrangement (schemes B1–B4) [21–26].
C. Balancing by using an auxiliary mechanism, which can be presented by three subgroups:
CI: Balancing by using an auxiliary linkage (schemes C1–C9) [27–33].
CII: Balancing by using a cam mechanism (schemes C10–C12) [34–36].
CIII: Balancing by using gear train (schemes C13–C14) [37,38].

The analysis of these studies showed that the balancing of robotic systems is carried out by weightless springs, i.e. the mass of spring is neglected. In most cases of balancing by using a cable and pulley arrangement such a approach is justified. The spring mass has not any influence on the balancing. However, for balancing by springs jointed directly with links of robotic systems or balancing by using an auxiliary linkage it is necessary to introduce the spring mass into balancing equations. Among several works, we can distinguish the study of Simionescu and Ciupitu [27], in which the spring mass is included in the balancing equations. However it is considered as a known parameter before the stiffness coefficient determination. It is obvious that by an iteration approach or by a preliminary estimation it is possible to obtain any approximate value of the spring mass and carry out a quasi-complete balancing of the mechanical system. However, especially for the rehabilitation devices, it might be as well to achieve a perfect balancing.

The organization of this paper is as follows: The next section describes the balancing equation of a rotating link with respect to the spring mass. Then the minimization technique is discussed, which takes into account the spring mass. Finally, an application such as an approach for balancing of rehabilitation device is also presented.

2. Improvement of balancing accuracy by taking into account the spring mass

The balancing of the gravitational forces of a link (1), which rotates around a horizontal axis, is schematically shown in Fig. 1. In this scheme, for weigh balancing a helical spring (2), jointed between a point A of the link and a fixed point B, is used. The movable co-ordinate axis system x_1Oy_1 attached to link 1 was chosen so that the point A is upon the Ox_1 axis.

The unbalanced moment can be expressed as follows:

$$M_{\mathrm{u}} = M_{\mathrm{g}} + M_{\mathrm{b}} \tag{1}$$

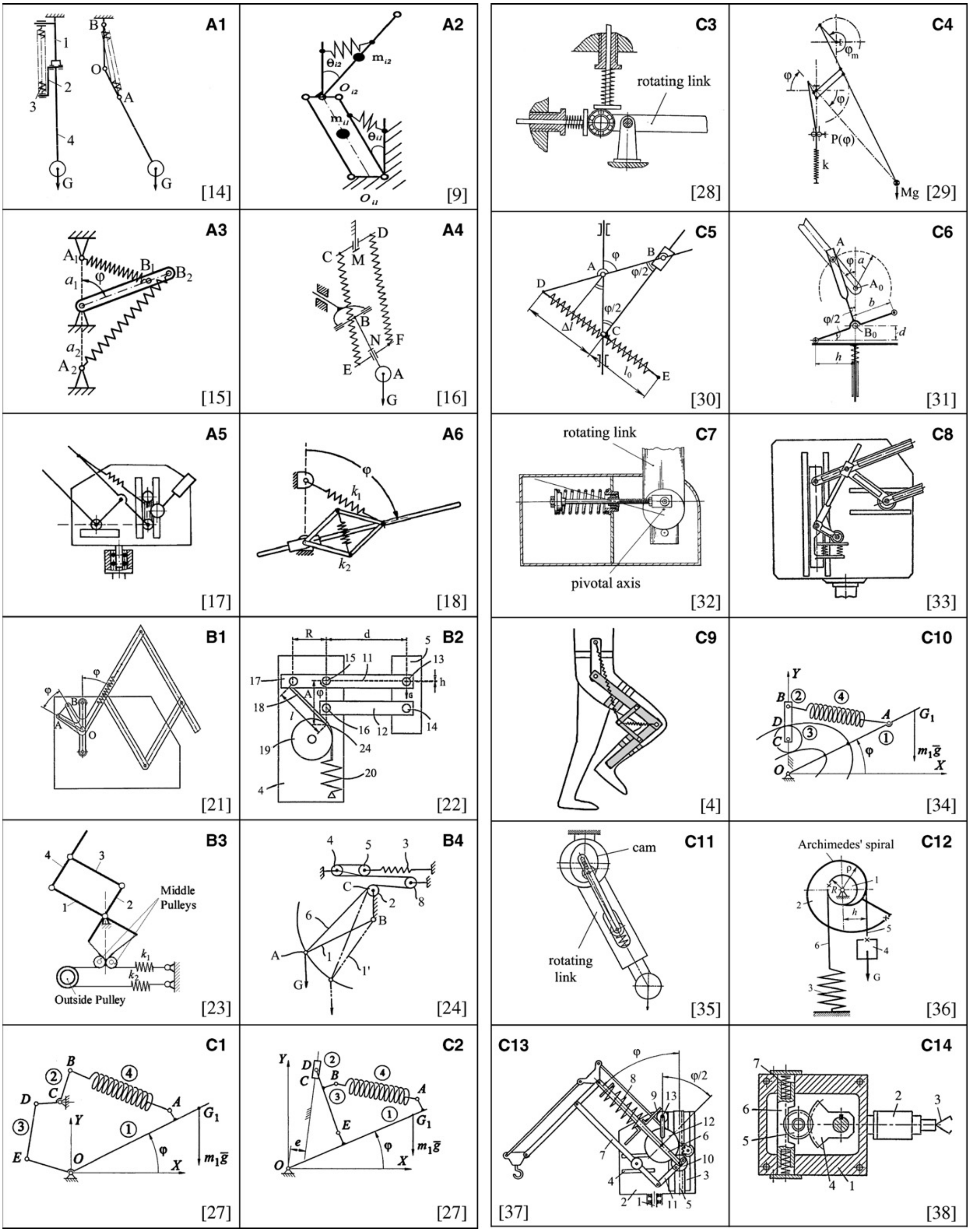
where M_{g} is the moment of the gravitational forces, M_{b} is the balancing moment of the elastic force of the spring.

For the cancellation of the unbalanced moment it is necessary to achieve the following condition:

$$[m_1 s_1 \sin(\varphi + \psi) + m_{2A} r \sin\varphi]g + F_S \frac{(X_B Y_A - X_A Y_B)}{l} = 0 \tag{2}$$

where m_1 is the mass of the rotating link, $s_1 = l_{OS_1}$ is the distance of gravity center S_1 from axis O, φ is the angle between Y-axis and x_1-axis, ψ is the angle between the axis x_1 and OS_1, m_{2A} is the concentrated point mass of the spring situated at the point $A(m_{2A} = m_2 s_2/l)$, m_2 is the mass of spring, $s_2 = l_{BS_2}$ is the distance of gravity center S_2 of the spring from point B, $l = l_{AB}$ the length of the spring at current angle φ, $F_S = F_0 + k(l - l_0)$ is the elastic force of the helical spring, l_0 is the initial length of the spring, F_0 is the initial force of the spring (the initial force is the internal force that holds the coils tightly together), k is the stiffness coefficient of the spring, $X_A = r\sin\varphi$, $Y_A = r\cos\varphi$, X_B and Y_B are the co-ordinates of the points A and B in the fixed co-ordinate axis system XOY, $r = l_{OA}$ is the distance of point A from axis O.

Table 1
Spring balancers



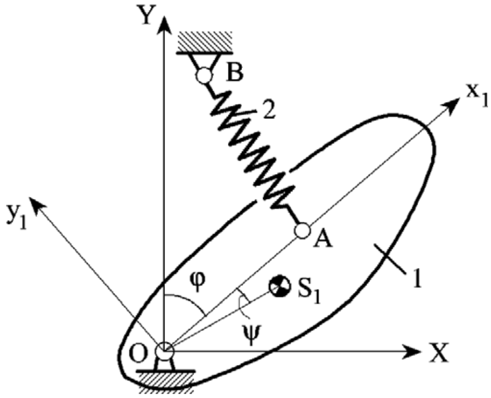


Fig. 1. Balancing of a rotating link.

The analysis of Eq. (2) shows that there are two solutions: (i) complete balancing when the zero free length spring is applied, (ii) partial balancing when the non-zero free length spring is applied.

2.1. Zero free length springs

The balancing of rotating link is favored by the use of zero free length springs, which is distinguished by the relationship $F_0 = kl_0$ or $F_0 = l_0 = 0$.

In this case, when $\psi = 0$ and $X_B = 0$, Eq. (2) leads to the following condition:

$$k = \frac{1}{Y_B r}\left(m_1 s_1 + m_2 \frac{r s_2}{l}\right) g \tag{3}$$

Considering that the extension of the spring is regular and $s_2 = l/2$, Eq. (3) can be written as

$$k = \frac{1}{Y_B r}(m_1 s_1 + 0.5 m_2 r) g \tag{4}$$

The stiffness coefficient and the mass of the spring also depend on its geometric and material parameters [39]

$$k = \frac{G d^4}{8 D^3 n} \quad \text{and} \quad m_2 = \frac{\rho L_w \pi d^2}{4} \tag{5,6}$$

where G is the Shear modulus calculated from the material's elastic modulus E and Poisson ratio v ($G = E/2(1 + v)$), d is the diameter of the wire that is wound into a helix, D is the mean diameter of the helix, n is the number of active coils, ρ is the material mass density, L_w is the length of wire.

From Eqs. (5) and (6) we can obtain the relationship between the mass of the spring and its stiffness coefficient

$$m_2 = \rho \pi L_w \sqrt{\frac{D^3 n k}{2G}} \tag{7}$$

On substituting this expression in (4), we obtain quadratic equation, from which we determine the stiffness coefficient

$$k = \left(-\frac{q}{2} \pm \sqrt{\left(\frac{q}{2}\right)^2 - u}\right)^2 \tag{8}$$

where

$$q = -\frac{\rho g \pi L_{\mathrm{w}}}{2Y_B}\sqrt{\frac{D^3 n}{2G}} \quad \text{and} \quad u = -\frac{m_1 s_1 g}{Y_B r} \tag{9, 10}$$

taking into account that

$$-\frac{q}{2} \pm \sqrt{\left(\frac{q}{2}\right)^2 - u} > 0 \tag{11}$$

Now let us consider the balancing by non-zero length springs.

2.2. Non-zero free length springs

There are two cases of the force-length characteristic of non-zero free length springs: with initial zero force ($F_0 = 0$) and non-zero initial force (taking into account that $F_0 \neq kl_0$).

In this case the complete balancing of a rotating link is impossible. Thus, an approximate solution may be applied.

For this purpose, we propose to minimize the root-mean-square (RMS) value of unbalanced moment

$$\mathrm{RMS} = \sqrt{\sum_{i=1}^{N}(M_{\mathrm{g}i} + M_{\mathrm{b}i})^2/N}, \tag{12}$$

where N is the number of calculated positions of rotating link.

For the minimization of the RMS, it is necessary to minimize the sum

$$\Delta = \sum_{i=1}^{N}(M_{\mathrm{g}i} + M_{\mathrm{b}i})^2 \rightarrow \min_k \tag{13}$$

or

$$\Delta = (C_1 + C_2\sqrt{k} + C_3 k)^2 \rightarrow \min_k \tag{14}$$

where

$$C_1 = \sum_{i=1}^{N} m_1 s_1 g \sin(\varphi_i + \psi) \tag{15}$$

$$C_2 = \sum_{i=1}^{N}\left(0.5\rho\pi g r L_{\mathrm{w}}\sqrt{\frac{D^3 n}{2G}}\right)\sin\varphi_i \tag{16}$$

$$C_3 = \sum_{i=1}^{N}\left(\sqrt{(X_B - r\sin\varphi_i)^2 + (Y_B - r\cos\varphi_i)^2} - l_0\right)\frac{X_B r\cos\varphi_i - Y_B r\sin\varphi_i}{\sqrt{(X_B - r\sin\varphi_i)^2 + (Y_B - r\cos\varphi_i)^2}} \tag{17}$$

For this purpose, we shall achieve the condition

$$\partial\Delta/\partial k = 0 \tag{18}$$

from which we obtain the following cubic equation:

$$z^3 + az^2 + bz + c = 0 \tag{19}$$

where

$$a = 3C_2/2C_3 \tag{20}$$

$$b = (C_2^2 + 2C_1C_3)/2C_3^2 \tag{21}$$

$$c = C_1C_2/2C_3^2 \tag{22}$$

$$z = \sqrt{k} \tag{23}$$

The solution of equation (19) with real coefficient can be expressed in algebraic form by means of Viette-Cordano method.

For determination of roots, first of all, we shall calculate

$$Q = (a^2 - 3b)/9 \tag{24}$$

$$R = (2a^3 - 9ab + 27c)/54 \tag{25}$$

When $R^2 < Q^3$, cubic equation has three real roots, determined by the following expressions:

$$z_1 = -2\sqrt{Q}\cos(t) - a/3 \tag{26}$$

$$z_2 = -2\sqrt{Q}\cos(t + 2\pi/3) - a/3 \tag{27}$$

$$z_3 = -2\sqrt{Q}\cos(t - 2\pi/3) - a/3 \tag{28}$$

$$t = \cos^{-1}\left[\left(R\sqrt{Q^3}\right)\Big/3\right] \tag{29}$$

When $R^2 \geqslant Q^3$, general cubic equation case has one real root and two real roots for confluent case.

For determination the complex roots, it is necessary to calculate

$$A = -\mathrm{sign}(R)\sqrt[3]{|R| + \sqrt{R^2 - Q^3}} \tag{30}$$

$$B = Q/A \ (\text{if } A \neq 0) \quad \text{and} \quad B = 0 \ (\text{if } A = 0) \tag{31}$$

The real root is

$$z_1 = A + B - a/3 \tag{32}$$

In the case, when $A = B$, the complex roots become the real roots

$$z_2 = -A - a/3 \tag{33}$$

After determination of z, we determine the stiffness of the spring (Eq. (23)) taking into account that $z > 0$.

3. Numerical examples and error analysis

For illustration of the suggested approach let us consider numerical examples. Numerical simulations were carried out for the balancing of the rotating link with following parameters: $m_1 = 8$ kg; $s_1 = 0.183$ m; $\psi = 0$ and $\varphi \in [2\pi/3;\ \pi]$. The parameters of the fixed points of the spring are the following: $Y_B = 0.16$ m and $r = 0.3$ m.

Firstly, this rotating link will be balanced by zero free length spring and then by non-zero free length spring.

3.1. Balancing by zero free length spring

The simple model of link balancing with weightless spring leads to the following parameters: $k = 299$ N/m; $F_0 = 121$ N.

Now let us balance the same link with zero free length spring taking into account its mass. The parameters which characterize the selected spring are the following: $G = 81{,}000$ N/mm^2, $\rho = 7800$ kg/m^3, $D = 0.04$ m, $n = 80$ and $L_w = 10.38$ m.

When the spring mass is included in the balancing equation we obtain $k = 324.5$ N/m and $F_0 = 131$ N. The mass of the spring is 0.8 kg.

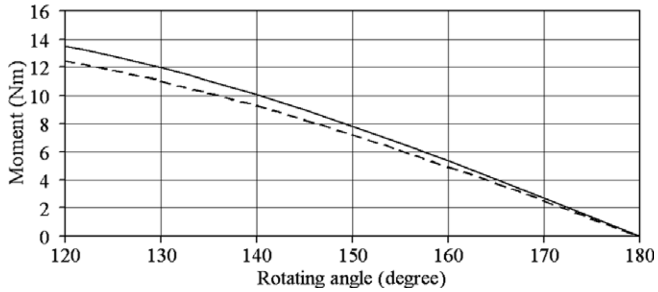


Fig. 2. Balancing moments for two examined cases: (- - -) first model with weightless spring; (—) second model with spring mass.

It should be noted that the parameters of the spring was selected, taking into account the admissible maximum extension and the spring index ($4 \leqslant D/d \leqslant 15$).

Fig. 2 shows the balancing moments for two examined cases. The error caused by neglect of the spring mass is 8.3%.

3.2. Balancing by non-zero free length spring

Now we consider the same problem with $F_0 = 0$ and $l_0 = 0.3$ m. The simple model of link balancing with weightless spring leads to $k = 990$ N/m. The spring with such stiffness and the selected geometric parameters (with $L_w = 8$ m) has 1.1 kg weight. This weight, which is neglected in the simple model add some unbalanced moment (Fig. 3).

Fig. 4 shows the theoretical values of the unbalanced moment and the effective values of this moment with spring mass.

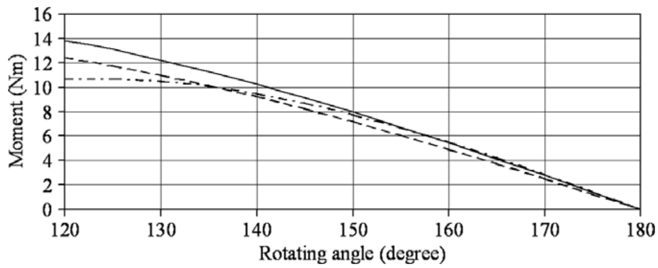


Fig. 3. Moment of the gravitational forces and the balancing moment of the spring elastic force: (- - -) moment of the gravitational forces for first model with weightless spring; (—) moment of the gravitational forces for second model taking into account the spring mass; (-··-) balancing moment with $k = 990$ N/m.

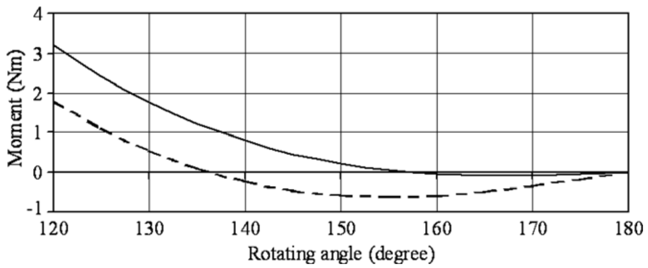


Fig. 4. Theoretical values of the unbalanced moment (- - -) and the effective values of this moment with spring mass (—).

The obtained results showed that the maximum value of the theoretical unbalanced moment is 1.81 N/m and the maximum value of the effective value (with spring mass) is 3.2 N/m.

Let us balance now the same link taking into account the spring mass. With the same parameters selected for our first example, from (15)–(17) we obtain $C_1 = 88.454$, $C_2 = 0.316$, $C_3 = -0.089$ and the following cubic equation:

$$z^3 - 5.3z^2 - 989.7z + 1748.8 = 0 \tag{34}$$

with $Q = 333$ and $R = -5.3$.

Thus $R^2 < Q^3$, we have three real roots determined by (26)–(29): $z_1 = 1.76$, $z_2 = 33.38$ and $z_3 = -29.84$. Taking $z = z_2$, we determine the stiffness of the spring $k = 1114$ N/m.

The spring with such a stiffness and the selected geometric parameters has 1.16 kg weight.

Fig. 5 shows the effective unbalanced moment due to the weights of the rotating link and spring, as well as the balancing moment of the spring with $k = 1114$ N/m.

Fig. 6 shows the effective values of the unbalanced moment when the mass of the spring was taken into account in the balancing equation.

Thus, in this case, the maximum value of the effective unbalanced moment is 1.86 N/m, which shows that significant improvement in balancing performance can be achieved through the use of the suggested approach.

It should be noted that the minimization of the function (14) was carried out by only one parameter: the spring stiffness. However this approach can be further optimized for spring connection points and other parameters of spring such as free length, i.e.

$$\Delta = (C_1 + C_2\sqrt{k} + C_3k)^2 \to \min_{k,X_B,Y_B,r,\psi,l_0} \tag{35}$$

The unknowns might be determined from the system of six equations obtained from the following conditions: $\partial\Delta/\partial\chi = 0$, $\chi = k$, X_B, Y_B, r, ψ, l_0. In this case, the solution cannot be achieved by analytical methods and the unknowns can be only determined by numerical investigation.

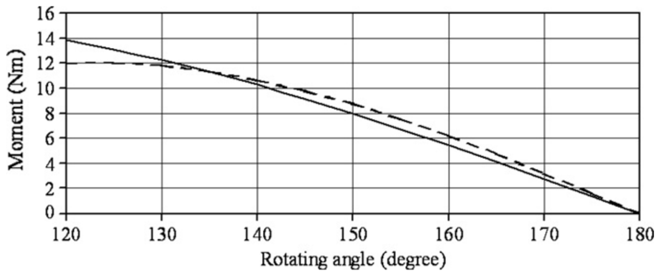


Fig. 5. Moment of the gravitational forces of the rotating link taking into account the spring mass (—) and the balancing moment of the spring with $k = 1114$ N/m (---).

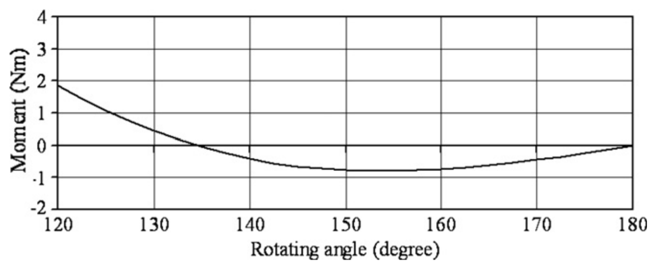


Fig. 6. Effective values of the unbalanced moment considering the spring mass.

4. Application to the balancing of leg orthosis for rehabilitation devices

Let us consider the design of a rehabilitation device, which can support the weight of leg during walking (Fig. 7). It is obvious that especially for the rehabilitation devices, it is hoped that the balancing will be perfect.

The following mass distribution and geometric parameters are considered for the leg [7]: $l_1 = 0.4322$ m, $l_2 = 0.421$ m, $m_1 = m_{\text{thigh}} = 7.39$ kg, $m_{\text{shank}} = 3.11$ kg, $m_{\text{foot}} = 0.97$ kg, $m_2 = m_{\text{shank}} + m_{\text{foot}} = 4.08$ kg, $l_{OS_1} = 0.41 l_1$, $l_{DS_2} = 0.44 l_2$. It should be noted that for two links design example m_2 consists of mass of the shank and the foot.

Let us consider the exact balancing of the leg with zero free length spring. For this purpose let us substitute mass m_1 of link 1 by two concentrated masses m_{1O} and $m_{1\text{D}}$ situated at the centers of joints O and D. Then we determine the common center of mass of the link 2 with concentrated mass $m_{1\text{D}}$

$$s_M = \frac{m_2 l_{DS_2}}{M} \tag{36}$$

with

$$M = m_2 + m_1 l_{OS_1}/l_1 \tag{37}$$

where s_M is the distance of the mass M from the center of joint D, l_{DS_2} is the distance of the center of S_2 from the center of joint D.

Thus, the masses of moving links are replaced by two masses: m_{1O}, which is fixed and M situated at the point s_M.

Now we connect a zero free length extension spring with the body at the point B and with the link 2 at the point A, and a compression spring with the body at the point O and with the link 2 at the point A. Please note that the point A coincides with the center of masses s_M.

Let us balance this system by considering the potential energy. The potential energy of the system can be written as

$$V = V_g + V_{S1} + V_{S2} \tag{38}$$

$$V_g = -[M + 0.5(m_{S1} + m_{S2})] g l_{OA} \sin\beta \tag{39}$$

$$V_{S1} = 0.5 k_{S1} l_{AB}^2 \tag{40*}$$

$$V_{S2} = 0.5 k_{S2} l_{OA}^2 \tag{41}$$

where $\beta = \angle XOA$, m_{S1} and m_{S2} are the masses of the springs, l_{OA} and l_{AB} are the distances between the corresponding points, $k_{S1} = k$ and $k_{S2} = -k$. Note please that k is the stiffness coefficient of springs, which is the same for both springs.

On substituting

$$l_{AB}^2 = l_{OB}^2 + l_{OA}^2 + 2 l_{OB} l_{OA} \sin\beta \tag{42}$$

in Eq. (40) and after Eqs. (39)–(41) in (38), we get

$$V = [k l_{OB} - Mg - 0.5g(m_{S1} + m_{S2})] l_{OA} \sin\beta + 0.5 k l_{OB}^2 \tag{43}$$

where l_{OB} is the distance of point B from axis O.

Thus, the potential energy becomes constant when the coefficient of $\sin\beta$ and l_{OA} is zero, i.e.

$$k l_{OB} - Mg - 0.5g(m_{S1} + m_{S2}) = 0 \tag{44}$$

This expression taking into account Eq. (7) can be rewritten as

$$Mg + 0.5g\pi\sqrt{k}\left(\rho_1 L_{\text{w1}}\sqrt{\frac{D_1^3 n_1}{G_1}} + \rho_2 L_{\text{w2}}\sqrt{\frac{D_2^3 n_2}{G_2}}\right) - k l_{OB} = 0. \tag{45}$$

[*] Note please that it is the potential energy for zero free length spring.

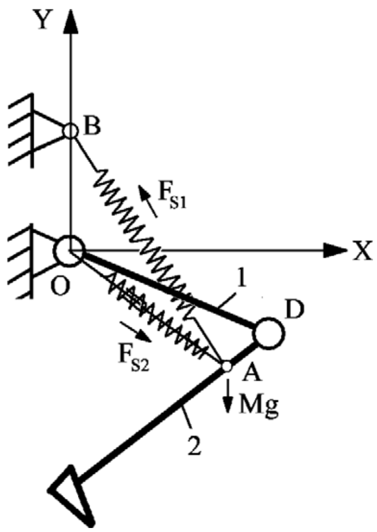


Fig. 7. Balancing device for the leg of a walking person.

Thus, by using two springs with parameters $G_1 = G_2 = 81000$ N/mm^2, $\rho_1 = \rho_2 = 7800$ kg/m^3, $D_1 = D_2 = 0.4$ m, $n_1 = 66$, $L_{w1} = 8.6$ m, $n_2 = 62$, $L_{w2} = 7.8$ m, we obtain $k_{S1} = 522$ N/m and $k_{S2} = -522$ N/m and the potential energy of the system becomes constant for all possible configurations, i.e. zero torques are required. In this way, an exact balancing of gravitational forces is achieved, which allows improving the balancing accuracy of rehabilitation devices.

5. Conclusion

The previous methods of gravity balancing of robotic systems presented in literature are limited by balancing through the use of weightless springs. In this paper, we had presented a new analytical approach to gravity balance considering the spring mass. For this purpose, the relationship between the stiffness coefficient of the spring and its mass is provided. Then this relationship is introduced into the balancing equation and the spring elastic force is determined by taking into account its mass. The both zero and non-zero free length spring designs were discussed. For zero length spring, the stiffness coefficient of the spring is determined from a quadratic equation and for non-zero length springs from a cubic equation. In the case of non-zero free length springs, the optimization was performed, which gave spring stiffness coefficient that minimizes the root-mean-square value of the torque at the joint.

The errors in gravity balancing due to the spring mass were also examined. It was shown that the mass of the balancing spring increases the unbalanced moment and it cannot be neglected. A numerical example was presented, which showed that the error caused by neglect of the spring mass can be reach until 8%.

In study [6] it was noted that the errors due to the practical restrictions and assumptions were 10–20%. We showed that the errors due to the spring mass are significant and for many balancing schemes the balancing accuracy can be increased by considering the spring mass.

A new balancing scheme for a rehabilitation device, which can support the weight of leg during walking, was proposed. It consists of two springs with the same stiffness coefficients, which are connected with the shank of the leg. It was shown an application of the improved balancing to the suggested system for a rehabilitation device. Future works concerning this study will be devoted to the development of a prototype and experimental verification of the obtained results taking into account not only the spring mass but also friction force.

Finally, we would like to note that many publications were mainly focused on the theoretical aspects of balancing. This paper sought to contribute practical considerations and it could be a useful tool for improvement of the balancing accuracy of robotic systems.

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