Kinematics, Workspace, Design and Accuracy Analysis of RPRPR Medical Parallel Robot

Cristian Szep†, Sergiu-Dan Stan†, *Member, IEEE,* Vencel Csibi†, Milos Manic‡, *Senior Member, IEEE,* Radu Bălan†

†Dept. of Mechatronics, Technical University of Cluj-Napoca, Romania, ‡Dept. of Computer Science, University of Idaho, USA

***Abstract* — In recent years, parallel robots find many applications in human-systems interaction, medical robots, rehabilitation, exoskeletons, to name a few. These applications are characterized by many imperatives, with robust precision and dynamic workspace computation as the two ultimate ones. Practical methods of kinematic's calibration make use of the linear differential error of the kinematics' model. This model is based on the Jacobian of the direct kinematics' model with respect to parameters of this model. The definition of the robot accuracy is usually related to robot positioning, so that the accuracy is defined as a measure of robot ability to attain a required position with respect to a fixed absolute reference coordinate frame. Such a definition is easily extended to trajectory tracking. Then, accuracy can be defined as a measure of robot ability to track the prescribed trajectory with respect to the absolute coordinate frame.**

***Keywords* — kinematics, workspace, design, Bipod parallel robot, RPRPR, 2 degrees of freedom.**

I. Introduction

In this times when the development of robot technology is increasing, people have a higher requirement for robot performance, not only high speed, high accuracy, but large workspace and low weight.

The architecture of these robots tends to reduce the positioning and orientation errors that appear in the industry of robots. In this paper it will be presented the design, kinematics and accuracy of a 2-DOF (Bipod) parallel robot (RPRPR).

First is presented the kinematic modelling of the studied robots, then a general modelling of errors in parallel robot chain is applied and generated for 2 DOF parallel robot.

The model is based on the use of error Jacobian matrices. By the error model, the end-effector positioning and accuracy can be more accurately estimated. Jacobian matrix was also used in obtaining errors.

The definition of the robot accuracy is usually related to robot positioning, so that the accuracy is defined as a measure of robot ability to attain a required position with respect to a fixed absolute reference coordinate frame. Such a definition is easily extended to trajectory tracking. Then, accuracy can be defined as a measure of robot ability to track the prescribed trajectory with respect to the absolute coordinate frame. The positioning errors of the end-effector have two principal origins:

- Lack of knowledge of the real robot geometry due to the manufacture tolerances and assembly errors of all its components.
- Some physical aspects such as the elasticity of links, the clearance in the joints and the temperature variations.

II. 2 Degree of Freedom Parallel Robot

A planar parallel robot is formed when two or more planar kinematic chains act together on a common rigid platform. The most common planar parallel architecture is composed of two RPR chains, where the notation RPR denotes the planar chain made up of a revolute joint, a prismatic joint, and a second revolute joint in series.

The planar 2 DOF parallel robot is shown in Fig.2. This structure is also known as 2-RPR robot. Since mobility of this parallel robot is two, two actuators are required to control this robot.

For simplicity, the origin of the fixed base frame {B} is located at base joint A with its x-axis towards base joint B, and the origin of the moving frame {M} is located in TCP, point P as shown in Fig. 1.

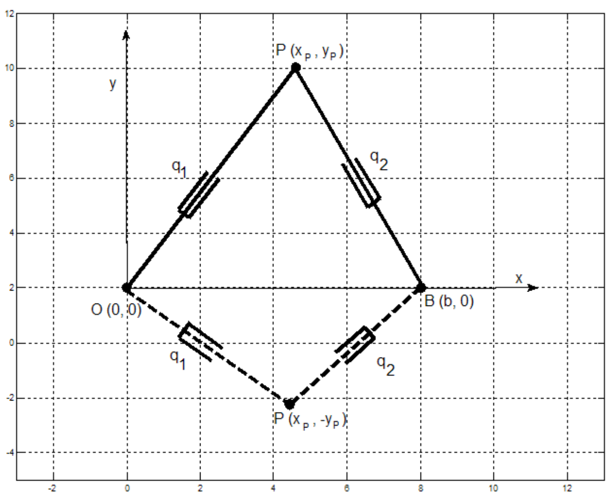

Fig. 1. Planar 2 DOF parallel robot.

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Dr. Sergiu-Dan Stan is with the Technical University of Cluj-Napoca, Dept. of Mechatronics, 103-105, B-dul Muncii, 400641, Cluj-Napoca, Romania (+40-264-401755; e-mail: sergiustan@ieee.org). Email contacts of other authors are: szep_crysty84@yahoo.com, csibivencel@yahoo.com, misko@ieee.org and radubalan@yahoo.com.

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The distance between two base joints is *b*. The position of the moving frame {M} in the base frame {B} is $\mathbf{x}=(x_P, y_P)^T$ and the actuated joint variables are represented by $\mathbf{q}=(q_1, q_2)^T$.

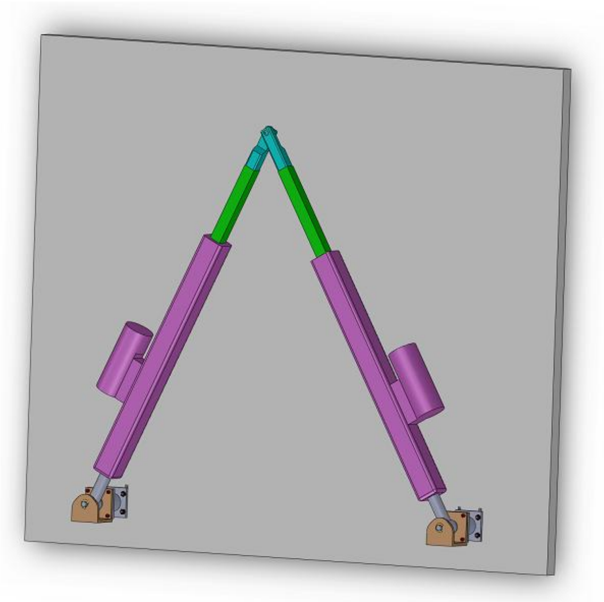

Fig. 2. CAD design of the 2 DOF parallel robot.

A. Kinematic analysis

Robot kinematics deal with the study of the robot motion as constrained by the geometry of the links. Typically, the study of the robot kinematics is divided into two parts, inverse kinematics and forward (or direct) kinematics. The inverse kinematics problem involves a known pose (position and orientation) of the output platform of the robot to a set of input joint variables that will achieve that pose. The forward kinematics problem involves the mapping from a known set of input joint variables to a pose of the moving platform that results from those given inputs. However, the inverse and forward kinematics problems of our parallel robot can be described in closed form.

The kinematics relation between **x** and **q** of this 2 DOF parallel robot can be expressed solving the:

$$f(\mathbf{x}, \mathbf{q})=0 \tag{1}$$

Then the inverse kinematics problem of the parallel robot can be solved by writing the following equations:

$$q_1 = \sqrt{x_P^2 + y_P^2}\,;\; q_2 = \sqrt{(b - x_P)^2 + y_P^2} \tag{2}$$

The TCP position can be calculated by using inverted transformation, from (6), thus the direct kinematics of the robot can be described as:

$$x_P = \frac{q_1^2 + b^2 - q_2^2}{2 \cdot b}$$
$$y_P = \sqrt{q_1^2 - x_P^2} \tag{3}$$

where the values of the x_p, y_P can be easily determined.

The forward and the inverse kinematics problems were solved under the MATLAB environment and it contains a user friendly graphical interface. The user can visualize the different solutions and the different geometric parameters of the parallel robot can be modified to investigate their effect on the kinematics of the robot. This graphical user interface can be a valuable and effective tool for the workspace analysis and the kinematics of the parallel robots. The designer can enhance the performance of his design using the results given by the presented graphical user interface.

The Matlab-based program is written to compute the forward and inverse kinematics of the parallel robot with 2 degrees of freedom. It consists of several MATLAB scripts and functions used for workspace analysis and kinematics of the parallel robot. A friendly user interface was developed using the MATLAB-GUI (graphical user interface). Several dialog boxes guide the user through the complete process.

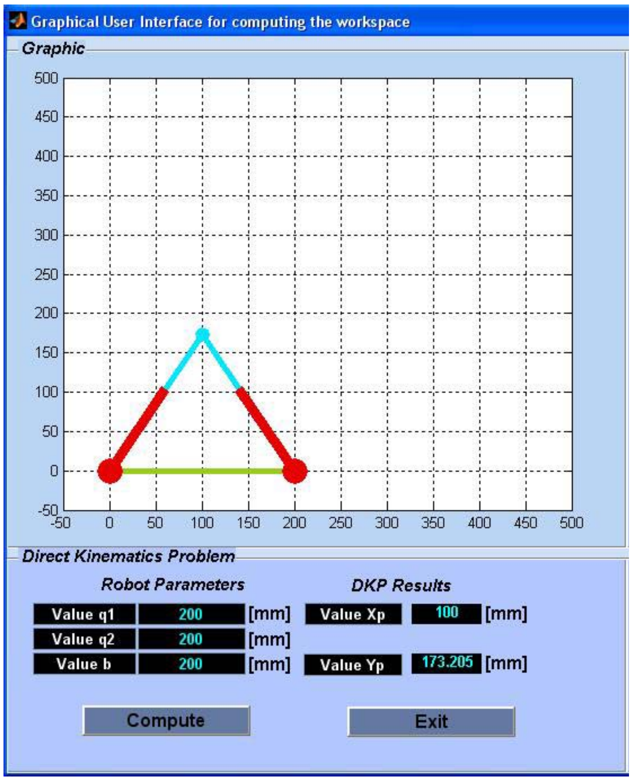

Fig. 3. Graphical User Interface (GUI) for solving direct kinematics of the 2 DOF planar parallel robot in MATLAB environment.

The user can modify the geometry of the 2 DOF parallel robot. The program visualizes the corresponding kinematics results with the new inputs.

B. Workspace

The workspace of a robot is defined as the set of all end-effector configurations which can be reached by some choice of joint coordinates. As the reachable locations of an end-effector are dependent on its orientation, a complete representation of the workspace should be embedded in a 6-dimensional workspace for which there is no possible graphical illustration; only subsets of the workspace may therefore be represented. There are different types of workspaces namely *constant orientation workspace*, *maximal workspace* or *reachable workspace*, *inclusive orientation workspace*, *total orientation workspace*, and *dextrous workspace*. The constant orientation workspace is the set of locations of the moving platform that may be reached when the orientation is fixed. The maximal workspace or reachable workspace is defined as the set of locations of the end-effector that may be reached with at least one orientation of the platform. The inclusive orientation workspace is the set of locations that may be reached with at least one orientation among a set defined by ranges on the orientation parameters. The set of locations of the end-effector that may be reached with all the orientations among a set defined by ranges on the orientations on the orientation parameters constitute the total orientation workspace. The dextrous workspace is defined as the set of locations for which all orientations are possible. The dextrous workspace is a special case of the total orientation workspace, the ranges for the rotation angles (the three angles that define the orientation of the end-effector) being $[0,2\pi]$.

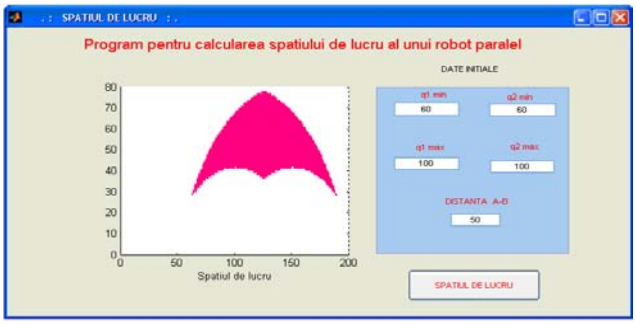

Fig. 4. The GUI for calculus of workspace for the planar 2 DOF parallel robot.

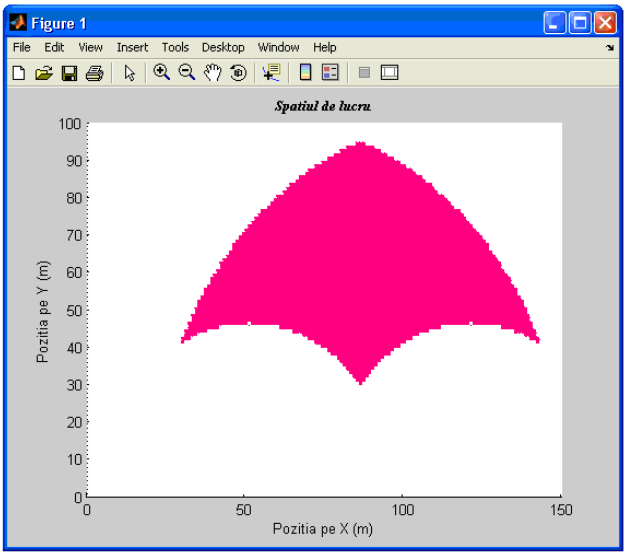

Fig. 5. Workspace of the 2 DOF parallel robot

C. Jacobian matrix

The general Jacobian matrix looks like this:

$$J = \begin{pmatrix} \frac{\partial k_1}{\partial q_1} & \frac{\partial k_1}{\partial q_2} & \cdots & \frac{\partial k_1}{\partial q_G} \\ \frac{\partial k_2}{\partial q_1} & \frac{\partial k_2}{\partial q_2} & \cdots & \frac{\partial k_2}{\partial q_G} \\ \frac{\partial k_3}{\partial q_1} & \frac{\partial k_3}{\partial q_2} & \cdots & \frac{\partial k_3}{\partial q_G} \end{pmatrix} \quad (4)$$

The Jacobian matrix of the 2 DOF parallel robot is:

$$J(q_1, q_2) = \begin{pmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} \\ \frac{\partial y}{\partial q_1} & \frac{\partial k_1}{\partial q_2} \end{pmatrix} \quad (5)$$

where:

$$\frac{\partial x}{\partial q_1} = \frac{q_1}{b}$$

$$\frac{\partial x}{\partial q_2} = -\frac{q_2}{b}$$

$$\frac{\partial y}{\partial q_1} = \frac{q_1 \cdot (b - x_p)}{y_p \cdot b}$$

$$\frac{\partial y}{\partial q_2} = \frac{q_2 \cdot x_p}{y_p \cdot b}$$

And the final Jacobian matrix looks like this:

$$J = \frac{1}{b} \begin{bmatrix} q_1 & -q_2 \\ \frac{q_1(b - x_p)}{y_p} & \frac{x_p q_2}{y_p} \end{bmatrix} \quad (6)$$

There are many possible sources of errors in a robot. These errors are referred to as "physical errors", to distinguish them from "generalized errors" which are defined later. The main sources of physical errors in a robot are:

· **Machining errors:** These errors are resulting from machining tolerances of the individual mechanical components that are assembled to build the robot.

· **Assembly:** These errors include linear and angular errors that are produced during the assembly of the various manipulator mechanical components.

· **Deflections:** Link and joint flexibility can cause elastic deformations of the structural members of the manipulator, resulting in large end-effector errors, especially in long reach manipulator systems. Local material deformations can also be another source of end-effector errors.

· **Measurement and Control:** Measurement, actuator, and control errors that occur in the control systems will create end-effector positioning errors. The resolution of encoders and stepper motors are examples of this type of error.

· **Joint errors:** These errors include bearing run-out errors in rotating joints and rail curvature errors in linear joints.

· **Clearances:** Backlash errors can occur in the motor gear box and in the manipulator joints.

In most cases, the physical errors are usually very small.

However, they can be amplified by the system to cause large errors at the end-effector. As a result, it is essential to identify those errors in the systems which significantly influence the end-effector positioning accuracy.

III. Error Modeling

Kinematic modeling and error modeling are established with all errors using Jacobian matrix method for the 3 link serial robot. In error analysis, error sensitivity is represented by the Jacobian matrix. The Jacobian approximation method is established. Using this method, error analysis, calibration, compensation, and on-line control model can be established.

In the next paragraph the Jacobian will be used to find the effect of errors in the actuators movements.

An error of **Δq** in actuators movement will produce a positional error of $\Delta x = J * \Delta q$.

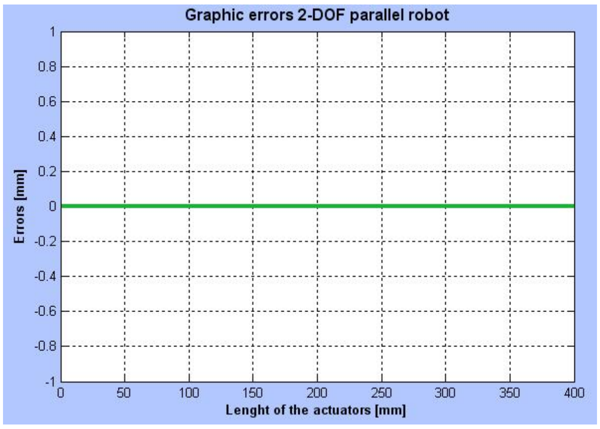

Fig. 6. Graphic errors 2-DOF parallel robot

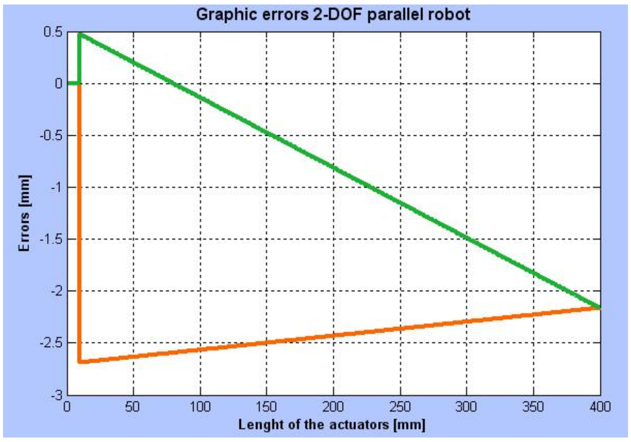

Fig. 7. Graphic errors 2-DOF parallel robot

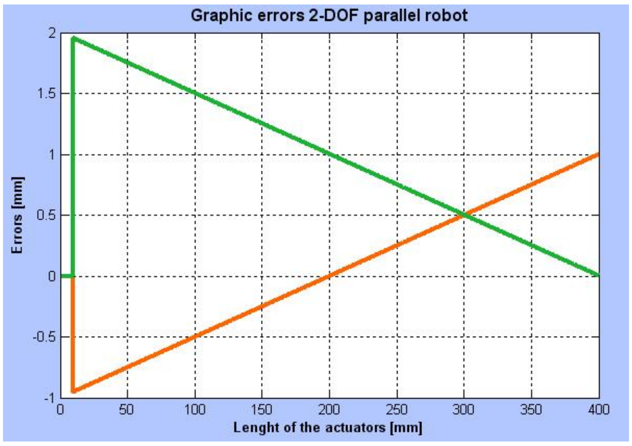

Fig. 8. Graphic errors 2-DOF parallel robot

The effect of errors will be different at different position, as shown in the next figures.

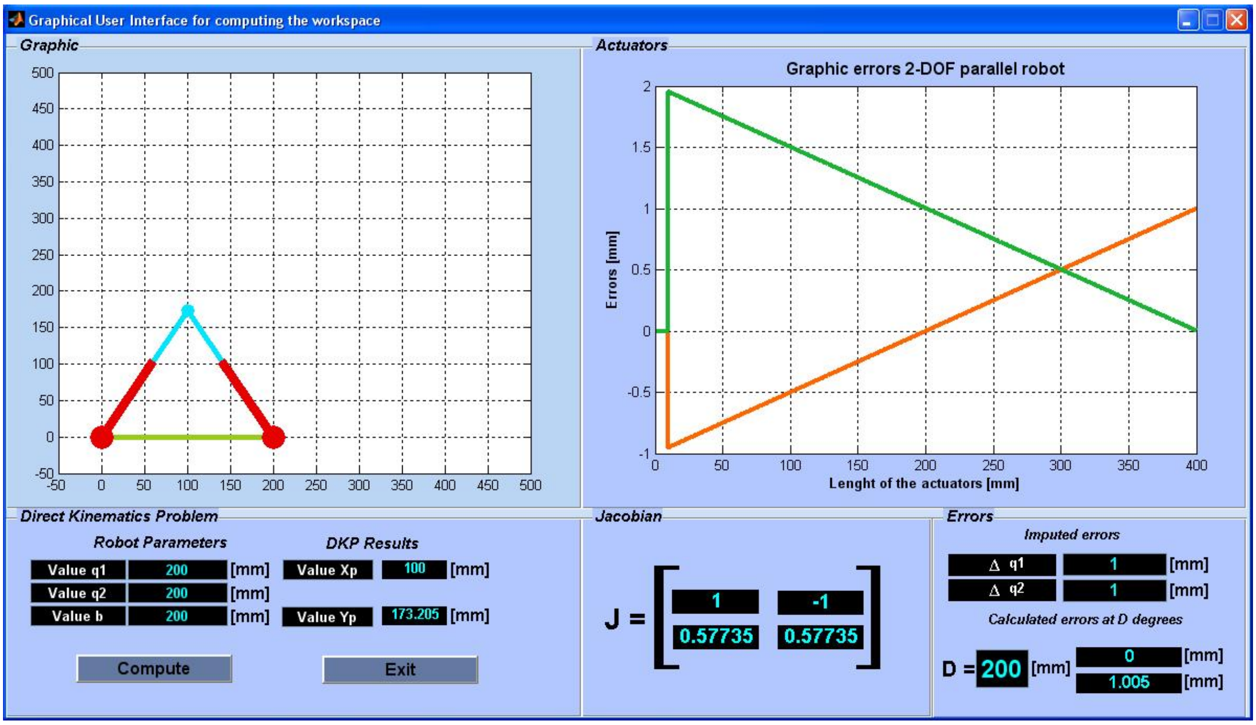

Fig. 9. Graphical User Interface for computing the Jacobian matrix, Direct Kinematics Problem (DKP) and errors of the 2-DOF parallel robot

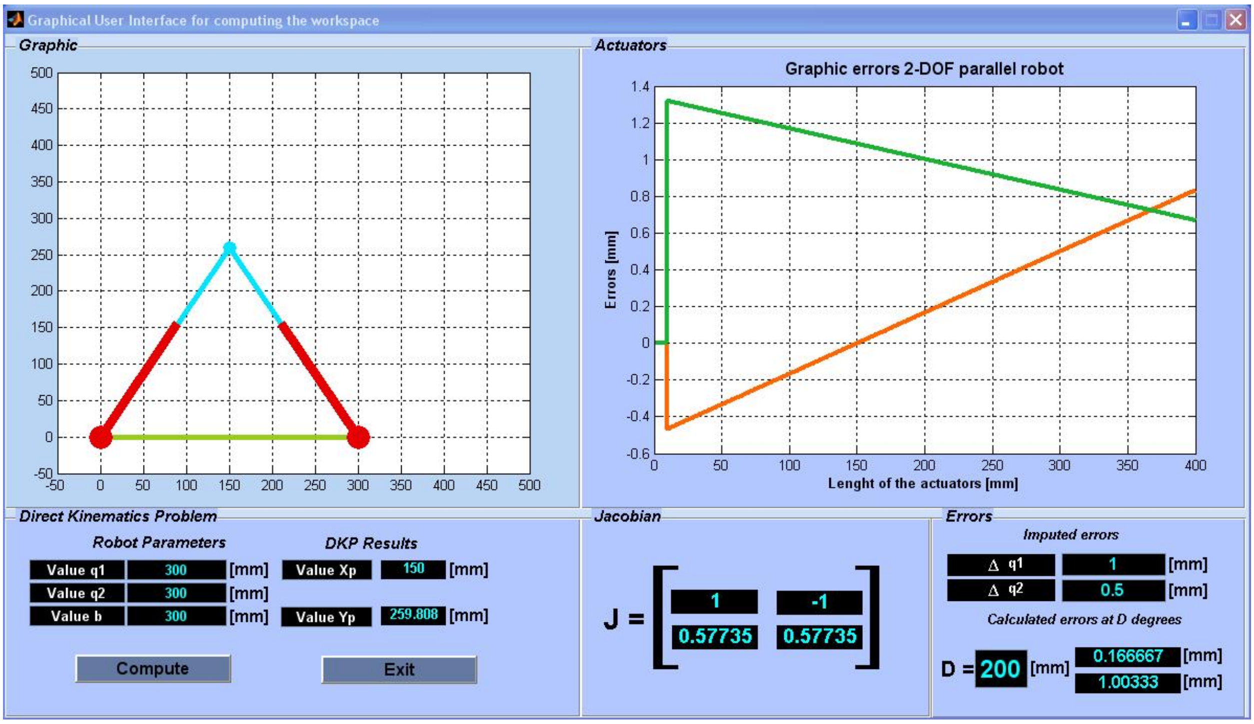

Fig. 10. Graphical User Interface for computing the Jacobian matrix, Direct Kinematics Problem (DKP) and errors of the 2-DOF parallel robot

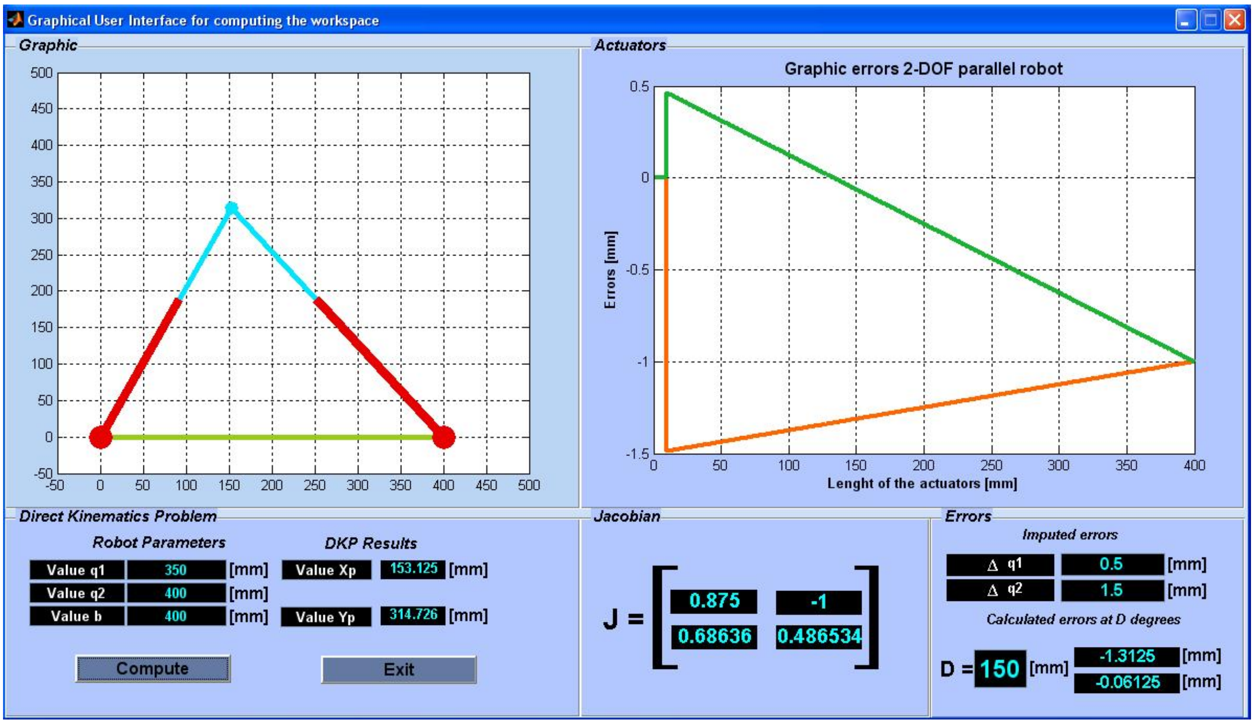

Fig. 11. Graphical User Interface for computing the Jacobian matrix, Direct Kinematics Problem (DKP) and errors of the 2-DOF parallel robot

The MATLAB-based program is written to compute the forward and inverse kinematics as well as Jacobian matrix value, and computed errors of the serial robot with 2 degrees of freedom.

Briefly, according to the present paper, it’s proposed a method and system for computing and modeling the errors of the end-effector of a 2 DOF robot.

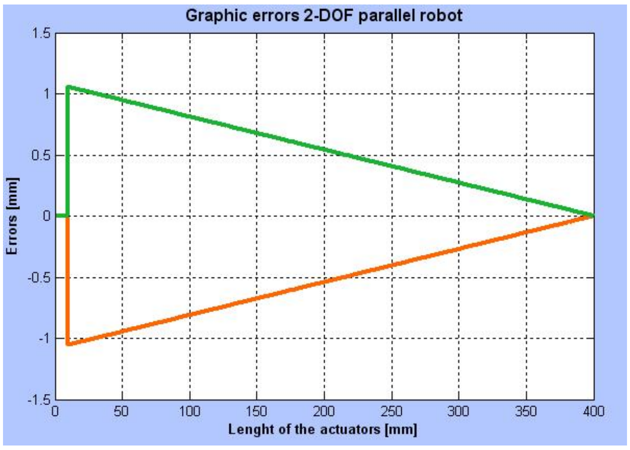

Fig. 12. Graphic errors 2-DOF parallel robot

The method is general and can be applied to any parallel robot. While it is based on classical concepts used in error analysis of mechanical systems, the method presented here, is formulated in a very simple and straight forward manner which makes it a practical solution for commercial applications and software development.

I. Conclusion

Parallel robots such in human-systems interaction such as medical, rehabilitation, exoskeleton robots depend on robustness, precision, and dynamic workspace computation, as the ultimate aspects of their safe and successful interaction with humans.

In this paper, the kinematic modeling and error modeling are established with all errors considered using Jacobian matrix method for the robot.

Based on the precision modeling and numerical simulations made on a 2 DOF parallel robot, it was drawn the following conclusions: accuracy modeling of robot errors and its application is presented in first part of the paper.

The definition of the robot accuracy is usually related to robot positioning, so that the accuracy is defined as a measure of robot ability to attain a required position with respect to a fixed absolute reference coordinate frame. Such a definition is easily extended to trajectory tracking. Then, accuracy can be defined as a measure of robot ability to track the prescribed trajectory with respect to the absolute coordinate frame.

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